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Development of a Statistical Model for Optimum Density in Hevea

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Production of latex depends on several factors and among those which have been shown to have significant influence are fertiliser levels and planting densities.

These two factors were studied together with an economic assessment to determine their optimum levels to ensure maximum return.

The model indicates that a combination of higher tree density and lower fertiliser level or lower tree density and higher fertiliser level will result in maximum return.

Investigation on optimum density and its interaction with fertiliser rates in perennial crops especially rubber is limited. However, some attempts were made in the investigation of the effect of various spacings on oil palm[1-3]. A number of studies on the effect of densities for *Hevea* have been published[4-6]. Recently, Iyer *et al.* studied the effect of densities based on systematic design developed by Nelder[7]. Iyer *et al.*[8] also fitted various models to the data. However, they did not take into consideration the earlier interaction effects between fertiliser level and density. This paper, therefore, attempts to develop a model which could incorporate the fertiliser effect (if any).

Duncan[9], in his paper, mentioned how the effect of nitrogen would change the optimum density required for maximum yield. The optimum density can be determined directly from the yield-density relationship function or the maximum difference between total cost and total sale functions. The latter is known as an economic optimum density.

EXPERIMENTAL DATA AND METHOD OF ANALYSIS

The data for the study were obtained from a systematic design on density trial for rubber. The experiment was conducted in Wessynton Estate, Kluang, Johore. The density was arranged in a systematic manner and conformed to the design suggested by Nelder[7]. The angle between the successive spoke was about 3.25° *(Figure 1)*. The data for analysis were taken from the average yield in grammes per tree per tapping over two years (details are available in Iyer *et al.*[8]). Two levels of fertilisers to simulate two fertility levels were introduced. The cumulative NPK used from planting up to two years of tapping was 4863 g per tree for the first level *(L1)* and 7996 g per tree for the second level *(L2)*. The ratio *L1:L2* was about 3:5. At planting time, level *L1* was applied with 220 g bowl sludge per hole and level *L2* was applied with 110 g CIRP per hole. Natural logarithmatic transformation was used on the data.

The density studied ranged from 247 trees to 884 trees per hectare. In standard commercial practice, the range of density is between 445 trees to 535 trees per hectare. If we accept the hypothesis that the relationship between natural logarithm of yield per plant per tapping and population is linear between 247 trees to 884 trees per hectare under the experimental conditions, several useful mathematical operations and statistical interpretations are available.

The formula for the relationship can be written as follows:

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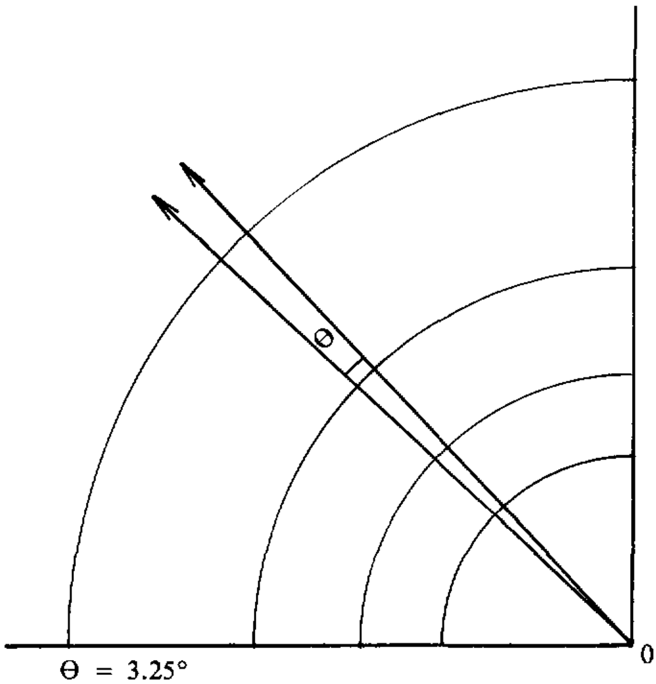

Figure 1. Systematic design: fixed shape, area increases with radius.

$$ln\ y = A + bx \text{ or } y = Ke^{bx} \quad ...1$$

where y = yield (g per tree per tapping)
A = constant
b = slope
x = plants per hectare
K = e^A.

It can be shown that the expected yield per hectare is the product of average yield (g per tree per tapping) and the number of plants per hectare. Thus the expected yield per hectare per tapping can be expressed as:

$$Y = x \cdot y \text{ or } Kxe^{bx} \quad ...2$$

By differentiating *Equation 2* with respect to x, we have the following solution:

$$\dot{Y} = Ke^{bx} + bKxe^{bx} \quad ...3$$

where Y = yield

$$\dot{Y} = \frac{dY}{dx}$$

By equating $\dot{Y}$ = 0 we can show that

$$x = \frac{-1}{b}$$

Applying the second degree of differentiation on *Equation 3* and substituting $x = \frac{-1}{b}$ into the final *Equation,* we can show that $x = \frac{-1}{b}$ is the optimum density under the experimental conditions. The maximum yield per tapping per hectare can be obtained by substituting $x = \frac{-1}{b}$ into *Equation 2*. Thus the maximun yield, *Ymax* can be expressed as:

$$Ymax = \frac{-K}{b} e^{-1} \text{ or } \frac{-K}{eb}; (b<0) \quad ...4$$

where e = 2.7183

(For example, the optimum density and the maximum average 1983/4 of PBIG seedling

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which has $K = 49.6501$ and $b = -0.131$ is x optimum = 763 trees per hectare, and *Ymax* = 13.94 kg per tapping per hectare respectively.)

The optimum density and maximum yield per tapping per hectare can be obtained directly from yield per plant, if any two densities are known. *Equation 1* can be written as follows:

$$ln\ (y_1) = a + bx_1$$
$$ln\ (y_2) = a + bx_2$$

where (x_1, y_1) and (x_2, y_2) are densities and yields of population 1 and 2 respectively.

$$ln\ (y_1) - ln\ (y_2) = b(x_1 - x_2)$$

$$b = \frac{lny_l - ny}{x_1 - x_2} \text{ or } \frac{ln\ (y_1/y_2)}{x_1 - x_2}$$

$$x(\text{optimum}) = \frac{-1}{b} \text{ or } \frac{x_l - x_2}{ln\ (y_1/y_2)} \quad \ldots 5$$

By expressing $y_1 = Ke^{bx_l}$ or

$$k = y_1e^{bx_1} ; b < 0 \quad \ldots 6$$

and substituting $b = ln(y_1/y_2)/(x_1 - x_2)$ into *Equation 6* we can obtain

$$Ymax = \frac{0.3678y_1\ (x_1 - x_2)}{e^{x^1(lny_1 - lny_2)/(x_1 - x_2)}(lny_1 - lny_2)} \quad \ldots 7$$

The *Equations 5* and *7* involve only two densities and their corresponding yields per tree per tapping.

The investigation of fertiliser effect and its interaction with density can be done by comparing the regression slopes and the intercepts between regression lines and the Y-axis. The comparison between slopes and between intercepts for PBIG seedlings and PR 261 are shown in *Table 1*. In this comparison, the difference between slope of fertiliser *L1* and slope of fertiliser *L2* in PBIG cannot be detected. The finding leads to reduction of regression lines to a single line as shown in *Figure 2*. The similar comparison for fertiliser effect for PR 261 shows that the comparison between slopes and between intercepts are significant at 1.0%. The effect is shown by two straight lines in *Figure 3*.

Fertiliser *L1* tends to reflect the effect more quickly on the lower densities (below 450 trees per hectare); whereas, fertiliser *L2* tends to reflect the effect more quickly on the higher densities *(Figure 3)*. It is shown in *Figure 4* that the maximum densities for PBIG, PR 261 *(L1)* and PR 261 *(L2)* based on *Equation 2* are 763 trees, 990 trees and 585 trees per hectare respectively.

Sometimes, the researcher is interested to know the best operating condition in a given experiment. Given knowledge of response function and specification of the goal and objective, the economic optimum operation can be estimated. Let us support that one has the following simple objective function.

$$P = Pyf(x) - Cpf(x) - Cx \quad \ldots 8$$

where P = profit

TABLE 1. COMPARISON OF REGRESSION FOR FERTILISER EFFECT FOR PBIG SEEDLINGS AND PR 261 BASED ON AVERAGE YIELD, 1983-4

Analysis of variance source	PBIG seedlings		PR 261	
	DF	MS	DF	t-value[a]
Between regression lines	2	0.00458 NS	—	—
Between slopes	1	0.00610 NS	12	3.515**
Between intercept	1	0.00306 NS	12	3.107**
Error	24	0.00632	—	—

[a]See *Appendix A* for calculation of t-value

**$P < 0.01$

NS = Not significant

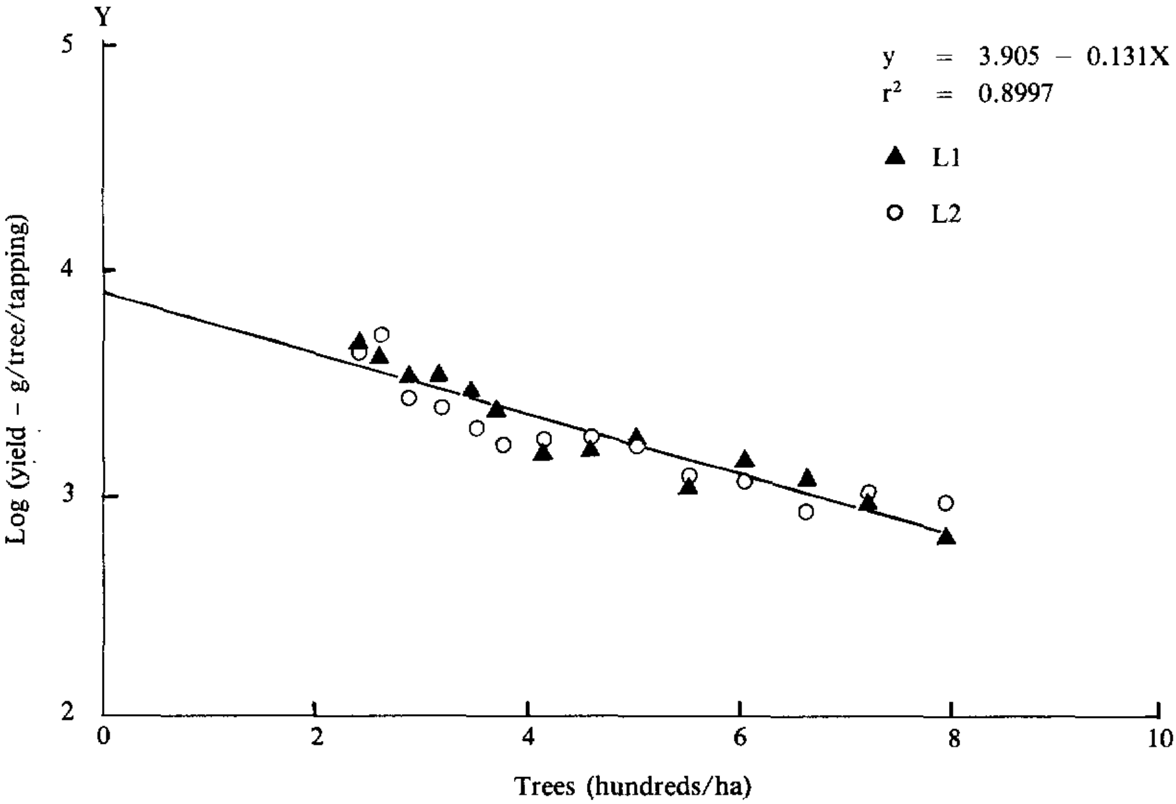

Figure 2. Fertiliser effect on 1983/4 average yield for PBIG seedlings.

Py = price per kilogramme

Cp = cost of production per kilogramme (mainly cost of tapping per kilogramme)

C = accumulative cost per tree before tapping

x = number of stand per hectare.

The fixed costs are ignored in the model and the yield is based on kilogramme per hectare per tapping. The function *f(x)* is the same as *Equation 2*.

The partial derivative of *Equation 8* with respect to x is:

$$\frac{\partial P}{\partial x} = (Py - Cp)\,\frac{\partial f(x)}{\partial x} - C$$

$$\text{Set } \frac{\partial P}{\partial x} = 0 \text{ then } \frac{\partial f(x)}{\partial x} = \frac{C}{Py-Cp} \quad \ldots 9$$

$$\text{or } \frac{C}{Py-Cp} = Ke^{bx}\,(1 + bx)\ ;\ b<0 \quad \ldots 10$$

The second partial derivative of *Equation 8* gives the following result:

$$\frac{\partial^2 P}{\partial x^2} = (2b + b^2x)\,Ke^{bx}\,(Py - Cp) \quad \ldots 11$$

Substituting *Equation 10* into *Equation 11* one can have the following result:

$$\frac{\partial^2 P}{\partial x^2} = bC + b(Py - Cp)\,Ke^{bx};b<0 \quad \ldots 12$$

Since *K, Py, Cp* and *Py − Cp* are greater than zero, *Equation 12* is always negative. This

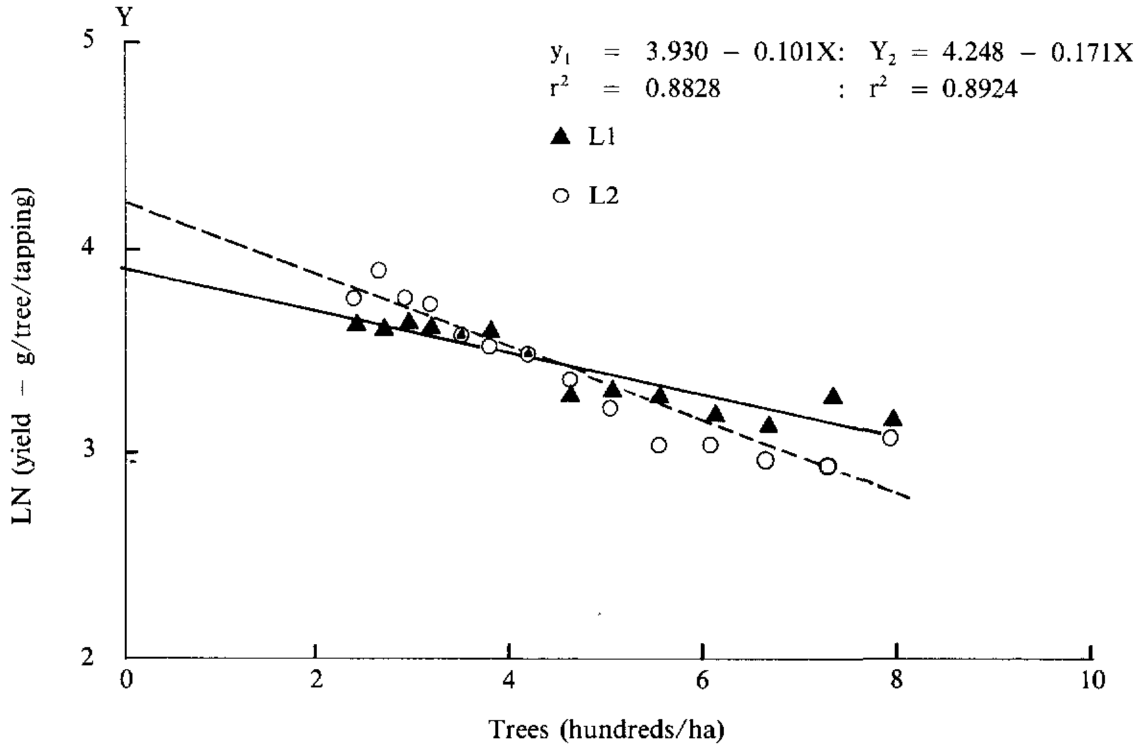

Figure 3. Fertiliser effect on 1983/4 average yield for PR 261.

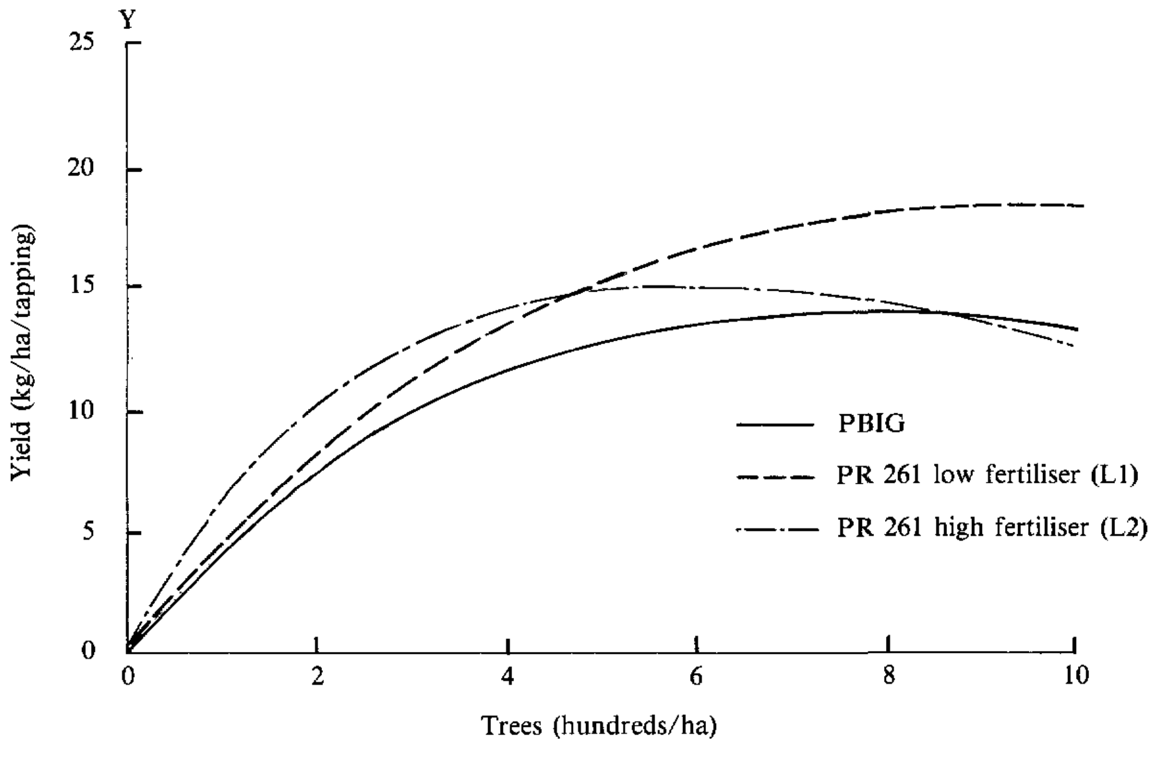

Figure 4. Relationship between yield and tree density for PBIG seedlings and PR 261.

indicates that the solution for x when $\frac{\partial P}{\partial x} = 0$ is optimum, and the value of P at that value of x is maximum.

By letting $\frac{C}{Py - Cp} = R$,

Equation 8 can be written as:

$$P = (Py - Cp)\,[f(x) - Rx] \qquad \ldots 13$$

Table 2 shows the optimal levels of x at different price ratios and *Table 3* shows how profit, P, changes as the density varies at $R = 8$ for PR 261.

In order to understand better the model and apply it in a real situation, the components of the costs in *Appendix B* are assessed. Using the information given in the appendix, the change in yield and profit as the density varies for PR 261 and PBIG are calculated *(Tables 4A and 4B)*.

RESULTS AND DISCUSSION

In general, it is impossible to consider all the input factors involved in producing a particular crop product. Perennial crops particularly rubber involve so many factors such as soil types, shapes of canopy, tapping systems, changes of weather from year to year, types of clone, planting densities, levels of fertiliser at the time of planting, frequencies of fertiliser application during the immature and mature period, topography of the area, *etc.* These factors in fact influence the production of latex. Therefore, one needs to simplify and select the factors in such a way that quantity of crop output is shown to be most responsive towards

TABLE 2. EXPECTED OPTIMAL DENSITIES AT DIFFERENT PRICE RATIOS FOR PBIG SEEDLINGS AND PR 261

Price ratio R	Densities (hundreds/ha)		
	PBIG seedlings	PR 261 (L1)	PR 261 (L2)
10	4.76	6.25	4.15
8	5.20	6.81	4.42
6	5.69	7.43	4.72
4	6.24	8.13	5.05
2	6.88	8.94	5.43
0	7.63	9.90	5.85

TABLE 3. EXPECTED PROFIT AT VARIOUS DENSITIES FOR PR 261 *(L1)* AND PR 261 *(L2)* AT $R = 8$

R (ratio)	Density (hundreds/ha)	L1		L2	
		Yield (kg/tapping/ha)	Profit (ringgit)	Yield (kg/tapping/ha)	Profit (ringgit)
8	1.50	6.57	5.57 (Py-Cp)	8.90	6.89 (Py-Cp)
8	3.00	12.07	9.27 (Py-Cp)	13.40	10.11 (Py-Cp)
8	4.50	14.46	10.86 (Py-Cp)	14.52	10.92 (Py-Cp)
8	6.00	16.50	11.70 (Py-Cp)	14.98	10.18 (Py-Cp)
8	7.50	17.97	11.97 (Py-Cp)	14.49	8.49 (Py-Cp)
8	9.00	18.33	11.13 (Py-Cp)	13.46	6.26 (Py-Cp)

TABLE 4A. EXPECTED YIELD AND PROFIT FOR DIFFERENT DENSITIES AT DIFFERENT FERTILISER LEVELS FOR PR 261

Density (hundreds/ha)	L1 Yield (kg/tapping/ha)	L1 Profit (ringgit)	L2 Yield (kg/tapping/ha)	L2 Profit (ringgit)
3	9.57	6.89	10.25	7.38
4	11.32	8.15	10.89	7.84
5	12.52	9.01	10.84	7.81
6	13.25	9.54	10.21	7.35
7	13.59	9.78	9.15	6.59
8	13.60	9.79	7.80	5.61
9	13.34	9.60	6.25	4.50

TABLE 4B. EXPECTED YIELD AND PROFIT FOR DIFFERENT DENSITIES AT DIFFERENT FERTILISER LEVELS FOR PBIG SEEDLINGS

Density (hundreds/ha)	L1 Yield (kg/tapping/ha)	L1 Profit (ringgit)	L2 Yield (kg/tapping/ha)	L2 Profit (ringgit)
3	8.35	6.01	7.63	5.50
4	9.48	6.83	8.53	6.14
5	10.05	7.24	8.86	6.38
6	10.16	7.32	8.73	6.29
7	9.91	7.13	8.24	5.94
8	9.38	6.75	7.47	5.38
9	8.62	6.21	6.48	4.67

the varying input quantities. The simple theory of crop response is that[10]:

- There is a continuous smooth relationship between inputs and output.
- The law of diminishing and decreasing returns to scale operates with respect to input factors.

The selection of the density per hectare as a single variable input factor for unconstrained objective function conforms with the above theory.

Fertiliser application at the time of planting might influence the yield production for a short period of time. While a change in the fertiliser level would cause a change in the optimum number of stands per hectare, the interaction effect between fertiliser and density depends on the types of clone and the fertiliser level. In this experiment the interaction between density and fertiliser in PBIG seedlings could not be detected, but a significant interaction was detected in PR 261.

The optimum density and maximum yield for a given fertiliser and clone cannot be drawn by merely a simple mathematical model without taking into consideration the materials used and the operating costs. An estate or smallholder need to operate within the optimum operation conditions at a given price and cost at a particular period in order to ensure that it can achieve the maximum profit. Thus, in any agronomic experiment, its economic analysis must be included to show its profitability. Maximum yield obtained at optimum input without considering costs would not always constitute an optimum condition. Thus, the estate or smallholder cannot enjoy the maximum profit.

Table 2 shows the optimal density at the price ratio of 8 for PR 261 *(L2)* with 442 stands per hectare. Using *Equation 4,* it is found that the optimum density is 585 stands per hectare. The former density yields a profit of 10.93 *(Py – Cp) (Table 3)* while the latter density yields a profit of 10.31 *(Py – Cp)* at the price ratio of 8. The latter profit is less because it does not operate at optimum condition. *Tables 4A* and *4B* show the profit variation at various densities for PR 261 and PBIG seedlings respectively. The two tables also provide an indication that a high density is more appropriate for a low fertiliser level than a high fertiliser level *(L2).* For example low fertiliser gives the actual optimum density of 585 stands per hectare while high fertiliser level gives the actual optimum density of 520 stands per hectare for PBIG seedlings *(Table 4B).* If the price ratio is within the range of 10 to 2, then the expected range of optimal density, as given by *Table 2,* will be between 625 trees to 894 trees per hectare for fertiliser level *L1* and 415 trees to 543 trees per hectare for high fertiliser level *L2* for PR 261.

A smallholder normally uses low fertiliser level, therefore, the high density is more appropriate, while for plantation which uses proper fertiliser the low density is appropriate.

The development of the model is, in fact, a part of modelling of agricultural production[11]. It is hoped that in future a simulation model for rubber industry based on the experimental data can be developed.

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APPENDIX A

Calculation of t-value for testing the methods of differences between two regression coefficients and between two intercepts when the two Mean Square Error (MSE) are not equal.

(a) Test for the difference between slope

$$\frac{b_1 - b_2}{\sqrt{\dfrac{\mathrm{MSE}_1}{D_1} + \dfrac{\mathrm{MSE}_2}{D_2}}}$$

where $D_i = \Sigma x^2_{ij} - n_{i\bar{x}}^2$

x_{ij} = the density of the ith level and jth observation

bi = regression coefficient at ith level

i = 1,2 ; j = 1,2 .. n

(b) Test for the difference between intercepts

$$\frac{a_1 - a_2}{\sqrt{\dfrac{\mathrm{MSE}_1}{R_1} + \dfrac{\mathrm{MSE}_2}{R_2}}}$$

where $R_i = \dfrac{(n_i D_i)}{\Sigma x^2_{ij}}$; $i = 1,2$

APPENDIX B

* Cost of CIRP at current price = 219.7 ringgit per tonne

** Average cost of budded stump, seedlings transportation and holing = 140 sen per plant

*** Tapping cost + other variable cost = 113 sen

Cost of 110 g CIRP = 2.2 sen

No cost incurred on 220 g bowl sludge.

Cost of fertiliser added before tapping:

- Low fertiliser *(L1)* = 268 sen per plant
- High fertiliser *(L2)* = 439 sen per plant

Total cost per plant for *L1* = 410.2 sen per plant

Total cost per plant for *L2* = 581.2 sen per plant

Assuming that the price of latex on the average is settled at 185 sen per kilogramme, let:

$Py = 185$ sen

$Cp = 113$ sen

$Py - Cp = 72$ sen

$R(Li) = C(Li)/(Py - Cp);\ i = 1,2$

$R(L1) = 5.69$

$R(L2) = 8.07$

where R is a price ratio.

*Price quoted for Johore State

**Estimate

***Based on Cost and Management Study