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Vulcanised Rubber Characterisation for Finite Element Analysis

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A Hyperelastic material model of vulcanised rubber compounds, defined as a strain energy function, is used to describe its mechanical behaviour. The relation of strain energy function to stress, following the study of Yeoh, was employed. The simplest relation of the reduced true stress for uniaxial tension or uniaxial compression (σ_r^t) and for simple shear (τ_r) is used as the following cubic function in (I_1–3):

$$\sigma_r^t \text{ or } \tau_r = 2C_{10} + 4C_{20}(I_1 - 3) + 6C_{30}(I_1 - 3)^2$$

where I_1 is the first invariant and C_{10}, C_{20} and C_{30} are constants, which are determined from those simple tests. Three vulcanised SBR compounds and one vulcanised NR compound were examined on those three simple deformation modes. An agreement of the experimental stress-strain relation to finite element analysis, using COSMOS/M was found for the four compounds tested in compression and simple shear modes. This agreement was also confirmed with other three NR compounds, tested in compression and simple shear modes. However, discrepancy was clearly observed in the case of high carbon-black loaded SBR compounds tested in tension mode.

Several theoretical models based on statistical thermodynamics and phenomenological approaches have been developed to characterise the mechanical behaviour of rubber[1,2]. The majority of the latest theories, including the one used in this study, assume rubber as an isotropic material and randomly orientated in the unstrained state. Stretching of rubber causes an orientation of the rubber molecules, but as the orientation is in the direction of stretching, the assumption of isotropy can be said to remain valid. This assumption is fundamental and is used to characterise rubber molecules by a quantity known as the strain energy function (W), which is the stored strain energy per unit volume.

Given that strain energy function is postulated in the phenomenological approach, the function is therefore determined *via* an experimental method. Numerous strain energy functions have been proposed. Such models explain W as a polynomial function of strain invariants, $W = W(I_1, I_2, I_3)$ which can be conversed in terms of principal stretch ratios, $W = W(\lambda_1, \lambda_2, \lambda_3)$, and whether incompressibility is assumed or not. I_1, I_2 and I_3 are the three invariants of the Green deformation tensor and can be defined in

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term of three principal stretch ratios, λ_1, λ_2, λ_3, as follows[1,2]:

- $I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$;
- $I_2 = (\lambda_1\lambda_2)^2 + (\lambda_2\lambda_3)^2 + (\lambda_3\lambda_1)^2$ and
- $I_3 = (\lambda_1\lambda_2\lambda_3)^2$.

The most frequently used *W* function, proposed by Rivlin[1], is shown in *Equation 1*. For incompressible condition of rubber, I_3 is equal to 1. This equation reduced to *Equation 2*, which is a power series of $(I_1–3)$ and $(I_2–3)$ and usually truncated to the first few terms. This study considers third-order term of *Equation 2* and follows two assumptions, proposed by Gregory (found in Charlton[2] and Yeoh[3]). These assumptions were $\partial W/\partial I_1$ which is much larger than $\partial W/\partial I_2$, and $\partial W/\partial I_1$ is I_2 independent. Therefore, the strain energy is written as *Equation 3* as shown below:

$$W = \sum_{ijk=0}^{n} C_{ijk} (I_1-3)^i (I_2-3)^j (I_3-1)^k \quad \ldots 1$$

$$W = \sum_{ij=0}^{n} C_{ij} (I_1-3)^i (I_2-3)^j \quad \ldots 2$$

$$W = C_{10} (I_1-3) + C_{20} (I_1-3)^2 + C_{30} (I_1-3)^3 \quad \ldots 3$$

where *i*, *j* and *k* are integers. C_{ijk}, C_{ij}, C_{10}, C_{20}, and C_{30} are constants, which must be evaluated from experiments. Gregory noted that a simple relationship existed between stress/strain data obtained in uniaxial tension, uniaxial compression, and simple shear. He showed a single curve for these test data when the reduced true stress was plotted against the invariant $(I_1–3)$[2,3], as the following equation:

$$\sigma^t_r \text{ or } \tau_r = 2C_{10} + 4C_{20} (I_1-3) + 6C_{30} (I_1-3)^2 \quad \ldots 4$$

where σ^t_r is the reduced true stress in uniaxial compression or tension, and τ_r is the reduced true stress in simple shear.

These stresses and $(I_1–3)$ can be expressed as a function of stress, strain and draw ratio as follows[2,3]:

For compression and tension,

$$\sigma^t_r = \frac{\sigma}{\lambda - \lambda^{-2}} = \frac{\sigma^t}{\lambda^2 - \frac{1}{\lambda}}$$

$$\text{and } (I_1 - 3) = \lambda^2 + \frac{2}{\lambda} - 3$$

For simple shear, $\tau_r = \frac{\tau}{\gamma}$

$$\text{and } (I_1 - 3) = \gamma^2$$

where σ and σ^t are engineering stress and true stress, respectively.

τ or γ are shear stress and strain, respectively.

PROCEDURE

This study is part of a research on 'bridge bearing computer modelling' using Finite Element Analysis (FEA) technique[4] and divided into two parts: the experiment and the computing; using the FEA package called COSMOS/M version 1.75 running on DOS. The experimental part includes taking data from literature study, and testing specimens following the *ASTM* and *BS* procedures[5–7]. The aim was to evaluate the average value of constants; C_{10}, C_{20} and C_{30} for each rubber compound. These constants will then be taken as material constants and input to the FEA program in order to analyse the stress-strain relation of the experimental deformation modes. The final results obtained from FEA will then be compared with the experimental results in order to confirm the correction of the material model used.

TABLE 1. EFFECTS AND LEVEL OF CARBON BLACK ON MODULUS AND HARDNESS OF RUBBER COMPOUNDS

Set	Compounds	Base polymer	Amount/Type of carbon black MT (p.h.r.)	HAF (p.h.r.)	E_0 (MPa)	Hardness (IRHD)
1	SBR 80	SBR	20	75	13.5[a]	80
	SBR 70	SBR	45	48	6.2[a]	70
	SBR 50	SBR	15	22	3.2[a]	50
	NR 45	NR	25	–	3.2[a]	45
2	BD 50	NR	–	40	3.3[b]	50
	BD 60	NR	–	60	5.4[b]	60
	BD 70	NR	–	70	7.7[b]	70

Note: [a]Youngs' modulus measured from tensile test at 50% strain
[b]Compressive modulus measured from uniaxial compressive test

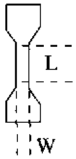
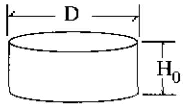
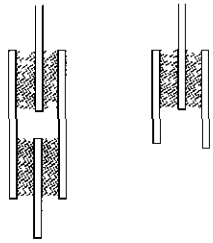
There are two sets of rubber compounds. The level of carbon black used and its effects on linear modulus and average hardness are shown in *Table 1*. The first set was the data from literature study[8], where the specimens have been tested on three modes of deformation: tension; compression and simple shear. Three compounds of carbon-black filled SBR of 50, 70 and 80 IRHD hardness and one carbon black filled NR compound of 45 IRHD hardness were studied, namely SBR 50, SBR 70, SBR 80 and NR 45, respectively. The second set was the carbon-black filled NR rubber having the average hardness of 50, 60, and 70 IRHD. The compounds of this set were moulded and tested in uniaxial compression and simple shear. They were formulated for elastomeric bridge bearings[4], and called BD 50, BD 60 and BD 70, respectively. Conditions of uniaxial tension, compression and shear tests for each compound are indicated in *Table 2*.

The strain-energy constants, C_{10}, C_{20} and C_{30} were then analysed, based on the relation in *Equation 4*. The best fitted equations from those experimental data were evaluated, using a least square fit method on the *Excel* worksheet. The results were then reported together with the correlation coefficient (r^2) of the fitted curve. 2D models of each compound deformed in tension, compression and simple shear, corresponding to their experimental conditions, were modelled. Conditions of each model are shown in *Table 3*.

RESULTS AND DISCUSSION

The reduced stresses plotted with (I_1 – 3) of three deformation modes: tension; compression and shear, of four compounds: SBR 50; SBR 70; SBR 80 and NR 45 are presented in *Figure 1(a)*. *Figure 1(b)* shows this relationship of BD 50, BD 60 and BD 70 for compression and shear test results. The best single line of the reduced stress is clearly shown in case of NR compounds (BD 50, BD 60, BD 70 and NR 45) and the lowest

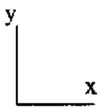
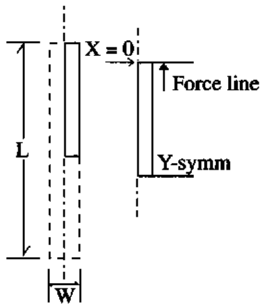
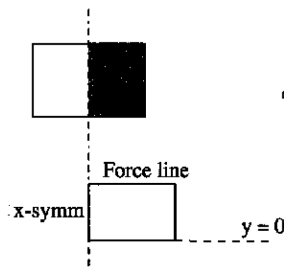
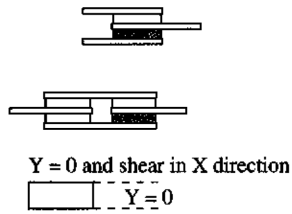
TABLE 2. CONDITIONS OF UNIAXIAL TENSION, COMPRESSION AND SIMPLE SHEAR TESTS

Conditions	Tension	Compression	Shear
Standard procedure	*ASTM D412*	*BS903 Part A14*	*BS903 Part A4*
Sample geometry	Dumbbell specimen 	Cylindrical specimen 	Double sandwich or sandwich specimen  (a) (b)
Size	$L \cong 32$ mm $W \cong 6$ mm Thickness $\cong 2$ mm	$D \cong 28$ mm $H_0 \cong 13$ mm	Area $\cong 25 \times 25$ mm^2 Thickness 5 mm
Cross-head speed	500 mm/min	5 mm/min	10 mm/min
Condition of testing	Cyclic test, taken at the 3rd loaded cycle		
Maximum strain	200%	$\cong$ 30–40%	$\cong 25\%^{(a)}$ and $60\%^{(b)}$
Others	Using Image Analysis Technique up to 200% strain	Lubricated at upper and lower surfaces	$^{(a)}$ For NR 45, SBR 50, SBR 70 and SBR 80 $^{(b)}$ For BD 50, BD 60, BD 70

filled SBR compound (SBR 50) but not for the other higher filled SBR. This could also be seen from a low r^2 value of the fitted line, evaluated for the average values of C_{10}, C_{20} and C_{30} (shown in *Table 4*). The two low r^2 values are 0.45 and 0.51 for SBR 80 and SBR 70 respectively, the two higher filled SBR compounds. This may be attributed to the higher non-linearity behaviour of SBR compared with the NR compounds.

The average percentage difference of stress (over the tested strain range) between experimental and FEA prediction is showed in *Table 5*. The stress-strain relationship obtained from the FEA compared with the experimental values is shown in *Figures 2* and *3*. *Figures 2(a)*, *2(b)* and *2(c)* show the comparison of 4 compounds: SBR 50; SBR 70; SBR 80 and NR 45, tested on tension, compression and shear, respectively. *Figures 3 (a)* and *3(b)* show the comparison of the

TABLE 3. CONDITIONS OF MODELLING

Conditions	Tensile model	Compression model	Shear model
Element group	2D-plane stress	2D-Axissymmetry	2D-plane strain
Boundary condition	Quarter model of extended area	Half model of compressed cylinder	Full size plane strain of one specimen
			
Mess size (mm × mm)	16 × 3	14 × 13	25 × 5
Element type	6 nodes triangle	6 nodes triangle	6 nodes triangle
No. of element	78	128	210
Load condition (applied displacement)	Max. U_Y = 40 mm 25 steps	Max. U_Y = –5.2 mm 20 steps	Max. U_X = 3.5 mm 20 steps
Max. applied % strain	250	40	40

3 compounds: BD 50; BD 60 and BD 70, for compression and shear, respectively.

The agreement of the comparison is found at the percentage difference of just about and lower than 20% in most cases, except for SBR 80 and SBR 70 tested in tension. This agrees with the low value of correlation coefficient of material constants evaluation, shown in *Table 4*. Failure of prediction might be caused by high carbon-black loading, which experience strain history effects. The maximum strain testing of each deformation was different in both sample geometry and strain history, especially in the case of tension which was tested up to 200% strain. The same maximum strain should be applied for every test.

CONCLUSION

The strain energy function and conditions according to the work of Gregory and Yeoh can be used to predict the behaviour of low carbon-black filled rubber compounds precisely. For non-extended application such as a bridge bearing compound, uniaxial compression and simple shear tests are satisfactory for the

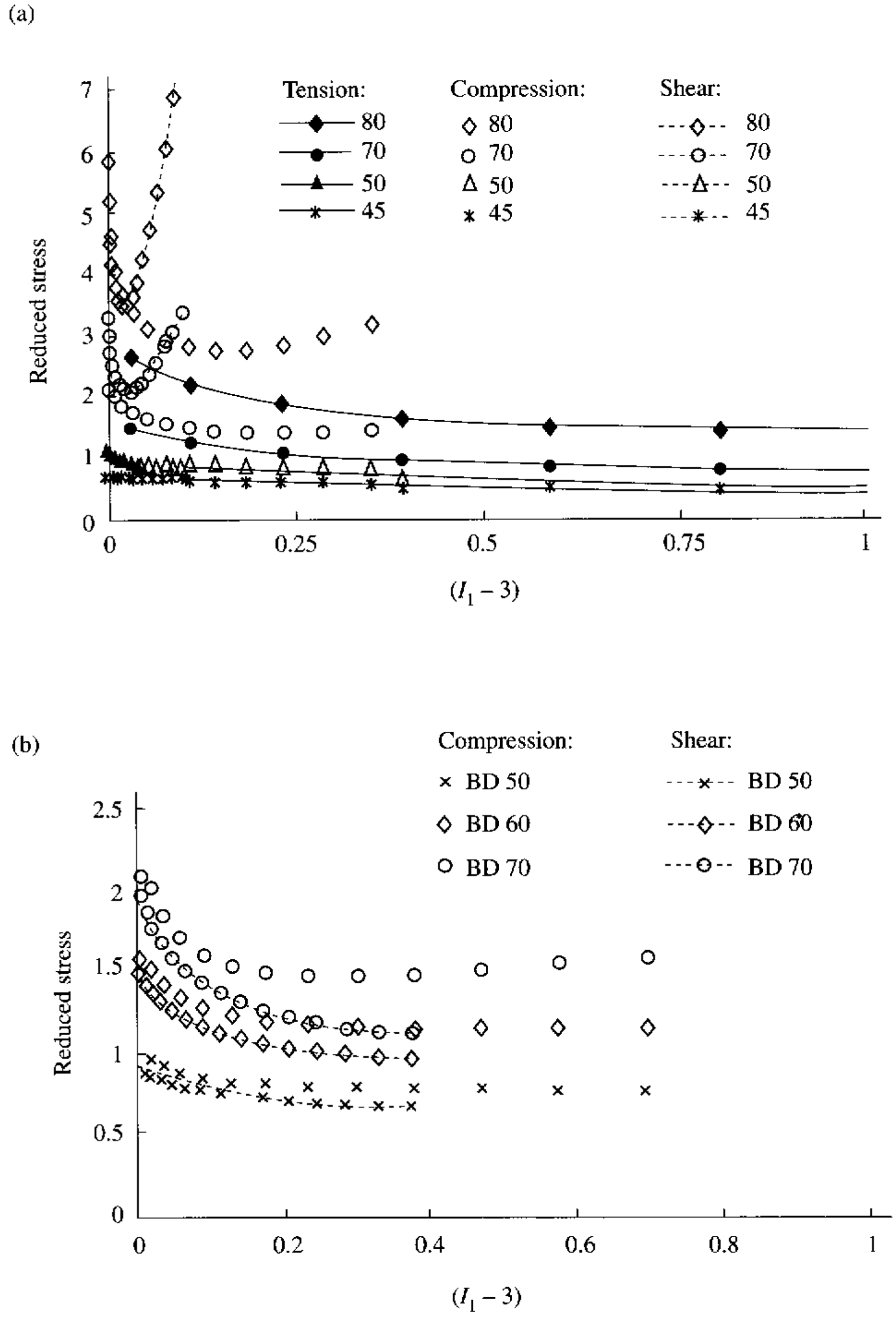
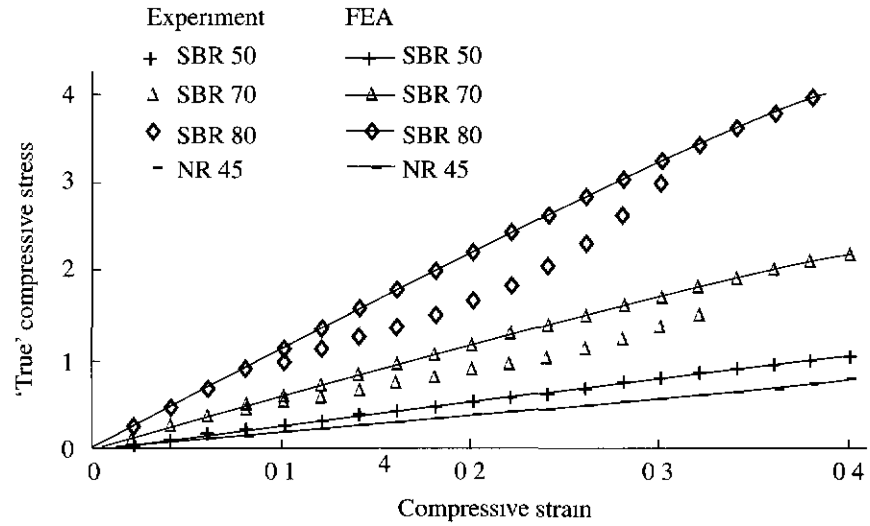


Figure 1. Reduced stress varying with ($I_1 - 3$) for (a) 4 compounds: SBR 50; SBR 70; SBR 80 and NR 45 tested in tension, compression and shear (b) 3 compounds: BD 50; BD 60, and BD 70 tested in compression and shear.

(b)

(a)



(c)

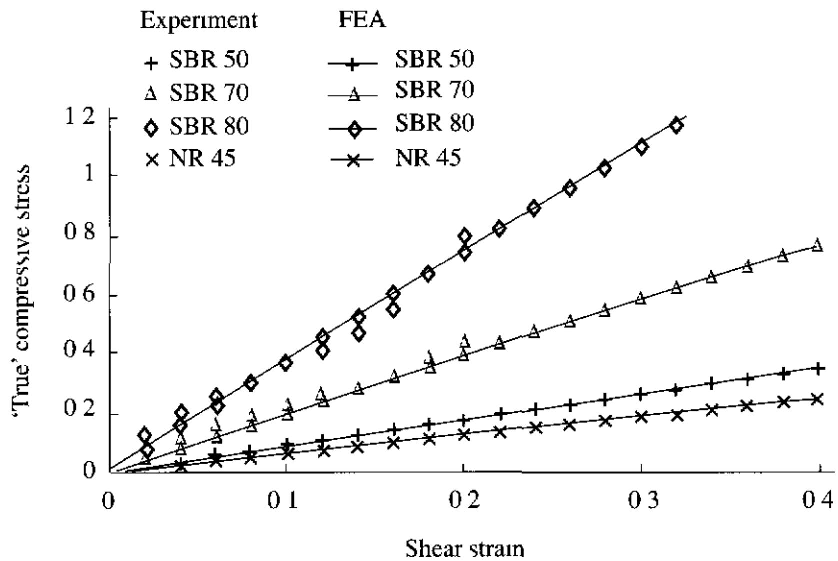
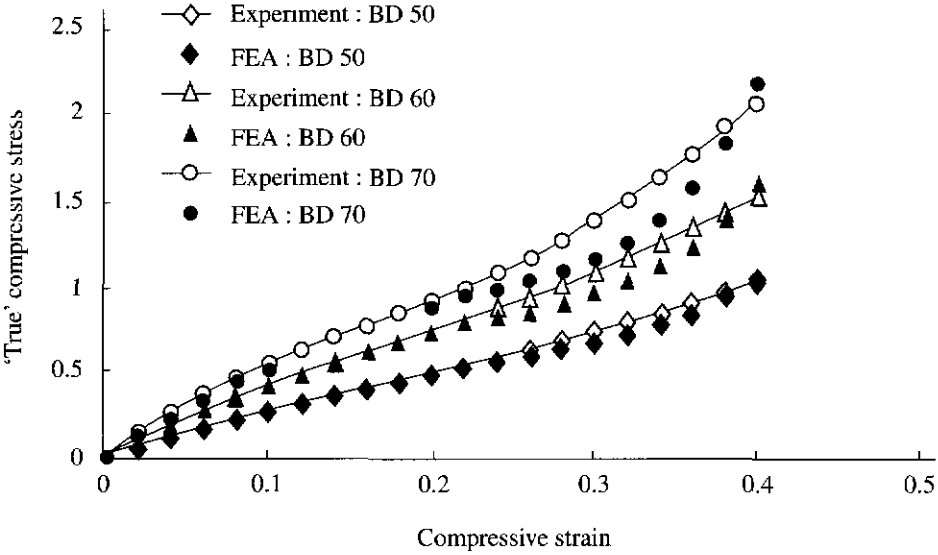


Figure 2 Comparison of stress strain relationship obtained from experiment and FEA from (a) tensile test (b) compression test and (c) shear test, for SBR 50, SBR 70, SBR 80 and NR 45

(a)



(b)

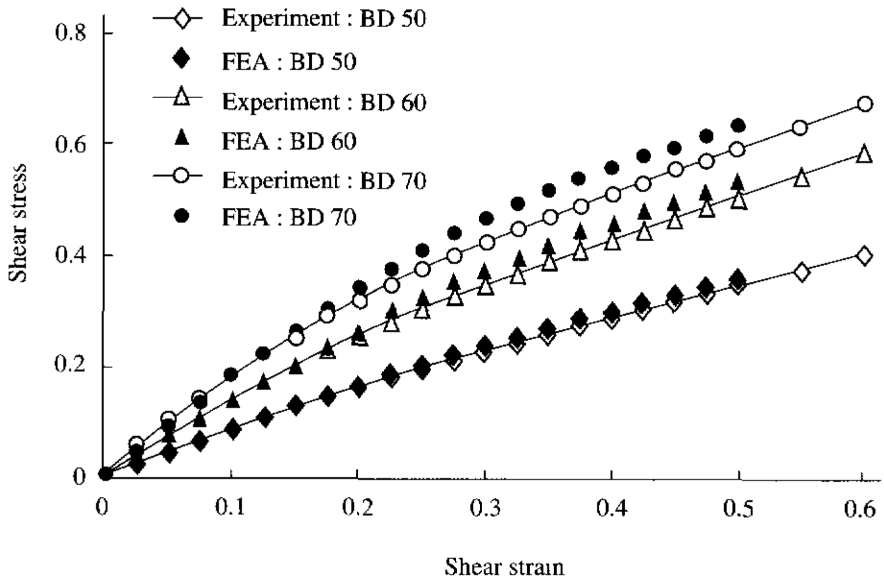


Figure 3. Comparison of stress-strain relationship obtained from experiment and FEA from (a) compression test and (b) shear test, for BD 50, BD 60 and BD 70.

TABLE 4. AVERAGE VALUE OF MATERIAL CONSTANTS (C_{10}, C_{20}, AND C_{30}) AND CORRELATION COEFFICIENT OF BEST FIT

Compounds/ Constants	SBR 80	SBR 70	SBR 50	NR 45	BD 70	BD 60	BD 50
C_{10} (MPa)	1.9126	1.0083	0.4553	0.3155	0.9282	0.7145	0.4464
C_{20} (MPa)	–0.2745	–0.1411	–0.0468	–0.0129	–0.8584	–0.5196	–0.2293
C_{30} (MPa)	0.0365	0.0189	0.0052	0.0023	0.8036	0.4495	0.2031
r_2	0.4545	0.5169	0.6926	0.8712	0.6884	0.7277	0.6245

TABLE 5. AVERAGE % DIFFERENCE OF PREDICTED STRESS COMPARED WITH EXPERIMENTAL STRESS OVER EXPERIMENTAL STRAIN RANGE

Compounds/ Test	SBR 80	SBR 70	SBR 50	NR 45	BD 70	BD 60	BD 50
Tension	43.83	37.82	12.89	6.55	–	–	–
Compression	18.40	22.22	6.20	2.26	–8.87	–6.33	–6.44
Shear	20.34	19.68	6.07	4.15	4.88	3.46	3.47

characterisation of rubber. The parameters provided from this step of characterisation will be able to predict the behaviour of products made with such compound. The prediction method for the behaviour of rubber products using the FEA technique provides an alternative way for rubber product design. This will be more economical than the conventional experimental method, which spend a longer time and entails higher cost.

Discrepancy tension mode testing of SBR compounds with carbon-black content higher than that of 50 p.h.r. (SBR 70 and SBR 80) was observed. However, good agreement is clearly seen from high black content of NR compounds (BD 60 and BD 70). Therefore, the behaviour of bearings made using these NR compounds (BD 50, BD 60 and BD 70) should be able to be predicted precisely, using the parameters, C_{10}, C_{20} and C_{30} provided from this method. A prediction of rubber bridge bearing is therefore a further aim of the study.

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