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# *Material Characterisation for Finite Element Analysis of Rubber Components*[†]

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*Experimental determination of the strain-energy function of an elastomer is in general not simple, even if simplifying assumptions are made of isotropy, incompressibility and perfect elasticity. This is because it requires simultaneous measurement of the stresses corresponding to imposition of two orthogonal strains which need to be varied independently over the full range The issue is often avoided; for example, many finite element packages recommend using stress-strain data from uniaxial tests to fit the constants in particular forms of strain-energy function. In consequence the usefulness of the finite element analyses can be compromised by the choice of an unsuitable strain-energy function. In the work presented here an intermediate technique, based on measurement of two stresses corresponding to a particular state of biaxial strain (shear) is used.*

*The technique is straightforward, althouth it requires a purpose built 'split pure-shear' straining jig. Although, it enables a more general form for the strain energy function to be derived, the results support a simplifying assumption which, if made, would permit the same information to be derived from a uniaxial test.*

*Strain-energy functions derived using the technique are given for natural rubber compounds with a range of loadings of N330 black and good agreement is found with uniaxial data. The technique is also useful for probing multiaxial relaxation and cyclic softening behaviour, and preliminary results on these aspects are reported.*

Commercial finite element packages treat rubber as a 'hyperelastic' material, *i.e.* a material that behaves elastically even for very large strains. In order to calculate the load-deformation behaviour of a rubber component and the strain distribution within it, whether by finite element analysis (FEA) or analytically, a 'constitutive model' is needed. For a hyperelastic material this model is expressed as a strain-energy function, giving the energy density in an element of material as a function of its state of strain.

Since the stress-strain behaviour for any type or magnitude of strain can be calculated from the strain-energy function, experimental derivation of the function would be expected to be an elaborate exercise. As FEA becomes widely used, there is a need to carry out this

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exercise on a routine basis for any rubber compound being considered for use in engineering components. This paper presents a relatively simple method of carrying out such characterisation, which is believed to be sufficiently general for most practical purposes.

## THEORETICAL BACKGROUND AND REVIEW OF EARLIER WORK ON CHARACTERISING RUBBER

Any homogeneous (*i.e.* uniform) deformation of a continuum may be resolved into a rigid body rotation and three principal stretches $\lambda_1$, $\lambda_2$, $\lambda_3$ along orthogonal axes (the principal axes). The principal stretches are the ratios extended to original length of lines embedded in the material oriented parallel to the principal axes, and are hence often called the extension ratios. The relationship of the principal stretches to the applied forces $f_i$ per unit unstrained area is, for an elastic material, completely determined by the strain-energy function, $W$. For an isotropic elastic material (a reasonable assumption for rubber) $W$ is a symmetric function of the $\lambda_i$:

$$W = W(\lambda_1, \lambda_2, \lambda_3) \qquad \ldots 1$$

and, as explained by Treloar[1]:

$$f_i = \frac{\partial W}{\partial \lambda_i} \qquad \ldots 2$$

On account of the 'hyperelastic' nature of the material the strains $(\lambda_1 - 1)$ in *Equations 1* and *2* cannot be regarded as infinitesimal as in classical elasticity theory.

For an incompressible material:

$$J \equiv \lambda_1\lambda_2\lambda_3 = 1 \qquad \ldots 3$$

and the true stresses $\sigma_1$ (*i.e.* forces per unit strained area) are indeterminant to the extent of a hydrostatic stress $p$, so that *Equation 2* becomes:

$$\sigma_1 = \frac{\lambda_1}{\lambda_1\lambda_2\lambda_3} f_1 = \lambda_1 \partial W/\partial \lambda_1 + p \qquad \ldots 4$$

According to the statistical theory of rubber elasticity[1]:

$$W = \frac{G}{2}\left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3\right) \qquad \ldots 5$$

where $G/2 = NkT$ with $N$ being the number of crosslinks per unit volume, $k$ is the Boltzmann's constant and $T$ the absolute temperature.

In the statistical theory, the rubber is assumed to be incompressible. *Equations 4* and *5* predict that the rubber force-deflection behaviour will be linear in simple shear even at high strains[1], and the shear modulus may be identified as the constant $G$. However, the predicted behaviour in uniaxial compression or tension is non-linear. *Equation 5* describes the elastic properties of unfilled vulcanisates quite well, and Rivlin[2] has shown it is the natural generalisation of linear, or Hookean, behaviour to finite strains, giving it the name 'neo-Hookean'.

The main departures from the predictions of *Equation 5* are:

(a) At very high strains the shear behaviour shows a rising rate, usually ascribed to finite extensibility of the polymer chains.

(b) If reinforcing filler is used, the shear stress-strain behaviour becomes non-

linear, showing a rapidly falling rate for small strains while the rising rate mentioned in (a) occurs at lower strains than for unfilled material, due to the strain amplifying effect of the filler

(c) The material is not perfectly elastic, especially if reinforcing filler is used

Much of the earlier effort was directed at improving the precision of modelling for unfilled rubber at moderate strains, since there are noticeable departures from *Equation 5*, rather than towards the grosser departures listed above With the advent of finite element analysis for engineering applications of rubber however, it is of considerable interest to meet points (a) and (b) This is because the local strains in a deformed component may be very large, even if the average strain is moderate, and because most engineering compounds contain reinforcing filler Point (c) is also of great relevance, but discussion of hysteresis will be postponed until later in the paper

If we wish to generalise *Equation 5* to meet points (a) and (b) it is not obvious how we should proceed Generally, commercial finite element software packages follow Rivlin[2] in re-writing *Equation 1* in terms of the strain invariants

$$W = W(I_1, I_2, I_3)$$

where $I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$

$I_2 = \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_1^2\lambda_3^2$

$I_3 = J^2 = \lambda_1^2\lambda_2^2\lambda_3^2$ 6

Isotropy is ensured since the strain invariants are symmetric functions of the $\lambda_1$, also, because the three strain invariants form an independent set, there is no loss of generality in re-writing *Equation 1* (for an isotropic material) in the form of *Equation 6*

$I_1/3$ represents the average factor by which the square of the length of embedded line segments, having random original orientation, increases The ends of such 'embedded line segments' take on a physical significance in the statistical theory as the crosslinks, assumed to move affinely, tying the long-chain molecules together[1] Similarly, $I_2/3$ represents the average factor by which the square of the area of a randomly oriented embedded surface element changes $I_3$ is equal to $J^2$ and thus the square of the factor by which the volume of an element changes in the deformation *Figure 1* shows the relationship of $I_1$ and $I_2$ for homogeneous deformations (*i e* states of uniform strain) for an incompressible material ($I_3 = 1$ so that, in *Equation 6*, $\lambda_3$ may be replaced by $1/\lambda_2\lambda_1$) We may regard $(I_1 - 3)$ as a generalised measure of strain, and $(I_2 - 3)$ as a measure (for constant $I_1$) of the character of the strain

Rivlin[3] showed that in terms of $I_1$ and $I_2$ *Equation 4*, for an incompressible material with $W = W(I_1, I_2)$, becomes

$$\sigma_1 = 2\left[\lambda_1^2 \frac{\partial W}{\partial I_1} - \frac{1}{\lambda_1^2}\frac{\partial W}{\partial I_2}\right] + p \qquad 7$$

Considering the case of a sheet of rubber having its surfaces force-free and normal to the z-axis, with the principal directions of strain within the sheet parallel to the x and y axes, it is apparent that

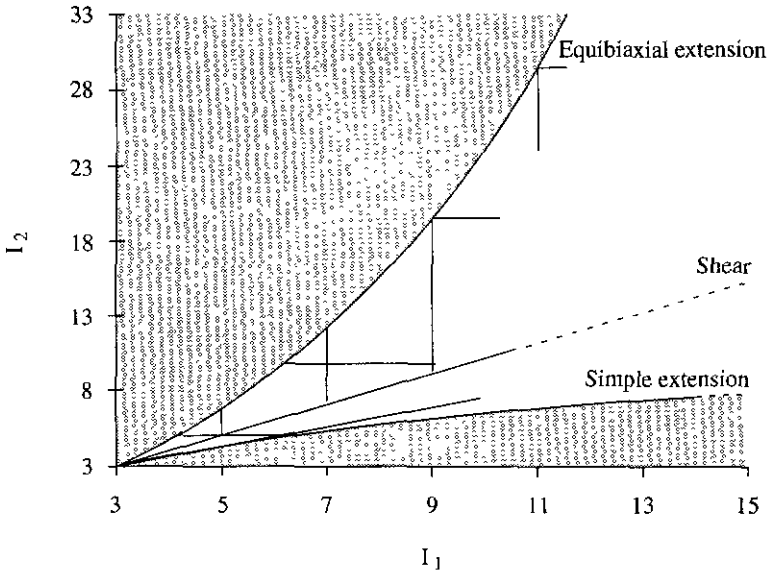



*Figure 1. Range of permissible values of strain invariants $I_1$ and $I_2$ for an incompressible material, shaded area shows impermissible values. Solid lines indicate values covered in the experiments of Rivlin and Saunders[4].*
*In shear, $I_1 = I_2 = \lambda^2 + 1/\lambda^2 + 1 = \gamma^2 + 3$ where $\gamma$ is the shear strain; in extension, $I_1 = \lambda^2 + 2/\lambda$, $I_2 = 2\lambda + 1/\lambda^2$; in equibiaxial extension $I_1 = 2\lambda^2 + 1/\lambda^4$, $I_2 = \lambda^4 + 2/\lambda^2$. In each case, $\lambda$ represents the largest of the extension ratios.*

$$\sigma_1 - \sigma_3 = \sigma_1 = 2\left(\lambda_1^2 - \frac{1}{\lambda_1^2\lambda_2^2}\right)\left(\frac{\partial W}{\partial I_1} + \lambda_2^2\frac{\partial W}{\partial I_2}\right)$$

$$\sigma_2 - \sigma_3 = \sigma_2 = 2\left(\lambda_2^2 - \frac{1}{\lambda_1^2\lambda_2^2}\right)\left(\frac{\partial W}{\partial I_1} + \lambda_1^2\frac{\partial W}{\partial I_2}\right) \quad \ldots 8$$

Solving the pair of simultaneous *Equations 8* gives:

$$\frac{\partial W}{\partial I_1} = \frac{\dfrac{\lambda_1^2\sigma_1}{\lambda_1^2 - 1/\lambda_1^2\lambda_2^2} - \dfrac{\lambda_2^2\sigma_2}{\lambda_2^2 - 1/\lambda_1^2\lambda_2^2}}{2\,(\lambda_1^2 - \lambda_2^2)}$$

$$\frac{\partial W}{\partial I_2} = \frac{\dfrac{\sigma_1}{\lambda_1^2 - 1/\lambda_1^2\lambda_2^2} - \dfrac{\sigma_2}{\lambda_2^2 - 1/\lambda_1^2\lambda_2^2}}{2\,(\lambda_2^2 - \lambda_1^2)} \quad \ldots 9$$

Rivlin and Saunders[4] pointed out that these equations are indeterminate for equibiaxial strain ($\lambda_1 = \lambda_2$), the two stresses then being equal, as confirmed by *Equation 8*. They also stated that any errors in $\sigma_1$ or $\sigma_2$ will be reflected in a greater percentage error in $\partial W/\partial I_1$ and $\partial W/\partial I_2$. In consequence, application of the equations is unreliable if the state of strain is (i) very small, so that both stresses are very small; (ii) close to uniaxial extension, so that

one of the stresses is very small, (iii) close to equibiaxial extension In their experimental work, the equations were successfully applied to a series of deformations such that the maximum value of $\lambda_1$ lay in the range $1.7 \leq \lambda_{max} \leq 2.8$ For each run they varied $\lambda_1$ and $\lambda_2$ in such a way that one of $I_1$ and $I_2$ was constant and the other one varied, these runs are shown as straight lines on *Figure 1* On account of the difficulty of applying *Equation 9* for values of $I_1$ and $I_2$ less than about 5 (equivalent to a simple shear strain of 141%) they supplemented these experiments with runs in simple extension, pure shear, a combination of simple extension and pure shear, equibiaxial extension and torsion with and without axial extension Each supplementary experiment yields a mixture of $\partial W/\partial I_1$ and $\partial W/\partial I_2$, but if simplifying assumptions are made (*e g* $\partial W/\partial I_1$ is constant), supported by the deductions from application of *Equation 9*, taken together they can enable $\partial W/\partial I_1$ and $\partial W/\partial I_2$ to be determined at smaller strains than is possible using *Equation 9* alone Rivlin and Saunders[4] finally concluded that for an unfilled natural rubber vulcanisate

$$W = C(I_1 - 3) + f(I_2 - 3) \qquad 10$$

where $C$ is a constant and $\partial f/\partial I_2$ is a decreasing function of $I_2$

Attempts by more recent workers[5,6] to apply *Equation 9* at small strain have led to confusing changes in the magnitude of $\partial W/\partial I_1$ and $\partial W/\partial I_2$ as the strain tends to zero

Application of the general strain approach of Rivlin and Saunders[4] to filled rubber is problematic because of the imperfect elasticity of the material In particular, the material shows hysteresis during a stress-strain cycle, and semi-permanent set and anisotropy afterwards James and Green[7] tried to reduce these effects by subjecting the filled rubber to ten large equibiaxial extensions before taking the detailed general stress-strain measurements However, the relevance of the results to components that have not undergone this procedure is then questionable It seems that carrying out a large number of different tests, as in the general strain procedure of Rivlin and Saunders[4], on filled rubber will raise as many questions as answers, and it is probably more fruitful first to investigate thoroughly the behaviour in a single mode of deformation on a virgin sample

Valanis and Landel[8] thought that the experimental determination of $W(I_1,I_2)$ proposed by Rivlin and Saunders[4] was so complicated, even for unfilled rubbers, that a simplifying assumption as to the form of $W$ should first be made They proposed the 'separable form'

$$W(\lambda_1,\lambda_2,\lambda_3) = w(\lambda_1) + w(\lambda_2) + w(\lambda_3) \qquad 11a$$

From *Equation 4* the differences in principal stresses of an incompressible material may be expressed as

$$\sigma_1 - \sigma_2 = \phi(\lambda_1) - \phi(\lambda_2) \; etc \qquad 11b$$

where $\phi(\lambda) \equiv \lambda w'(\lambda) - w'(1)$ and $w'(\lambda) \equiv dw/d\lambda$ Hence, for a pure shear experiment where $\lambda_2 = 1$, the value of $\phi(\lambda)$ may be found provided both $\sigma_1$ and $\sigma_2$ are measured From $\phi(\lambda)$, stresses in any other homogeneous deformation can be calculated using *Equation 11b*

This restricted form for $W$ has been found to work well in practice, at least for unfilled rubber up to moderately large strains[5,8,9]

The technique proposed in this paper is a particular way of carrying out the pure shear experiment with measurement of both non-zero stresses The interpretation of the results need not make the assumption of *Equation 11*, although this is an option By choosing this simplified technique the need for building a complicated apparatus, capable of applying a completely general biaxial strain to a test piece and measuring the two strains and stresses, is avoided The procedure for operating the instrumented pure shear apparatus is also much easier to define since there is only one variable — the largest principle strain For materials with strain history effects this reduction in versatility may be seen as an advantage rather than a disadvantage, at least for an initial study

## MATERIALS AND METHODS

### Apparatus

An apparatus for applying a pure shear deformation to rubber, enabling measurement of the force developed in the clamps and preventing departure of the lateral extension ratio of the rubber from unity, was designed As depicted in *Figure 2*, the clamps along one side of the test piece were split 50 mm from each of the free ends The split pieces were attached through parallel cantilever load cells to the central portion of the clamp These load cells were chosen because their output should be insensitive to normal load (*i e* the direction in which the test piece is stretched) or couples

It is important that the rubber in the region between the slits is in a state of pure shear Then, if the compliance of the load cells and the distortion of the strain field caused by the presence of the slits is negligible, each load cell will by symmetry measure half of the force ($F_2$) needed to constrain the value of $\lambda_2$ at unity By averaging the outputs from the load cells, the effect of any variation (such as height, thickness or elastic properties) across the test piece width will be averaged, as it is for the force measured in the longitudinal direction Moreover, any departure of the main straining direction from precise orthogonality to the bottom clamp will produce opposite contributions to the outputs in the side load cells, and so errors from this source will largely be cancelled out when the average output is taken

The test pieces were moulded with bonded metal strips to make the overall dimensions, clamping conditions and slit dimensions more reproducible

### Tension and Compression Experiments

In addition to pure shear tests, experiments were carried out in simple extension and in compression Tension was carried out on strips of rubber (180 mm × 20 mm × 1 mm) cut from one of the mouldings formed in the three-cavity mould used for the pure shear test piece Compression was carried out on cylindrical test pieces (32 mm diameter and nominally 25 mm thick) lubricated with detergent and placed between smooth flat plates, carefully set up to be parallel

### Materials

Experiments were carried out on the natural rubber vulcanisates given in *Table 1* The formulations, differing only in content of black and process oil, are taken from the Engineering Data Sheets[10], wherein comprehensive results for standard physical tests and dynamic shear stress-strain behaviour are given

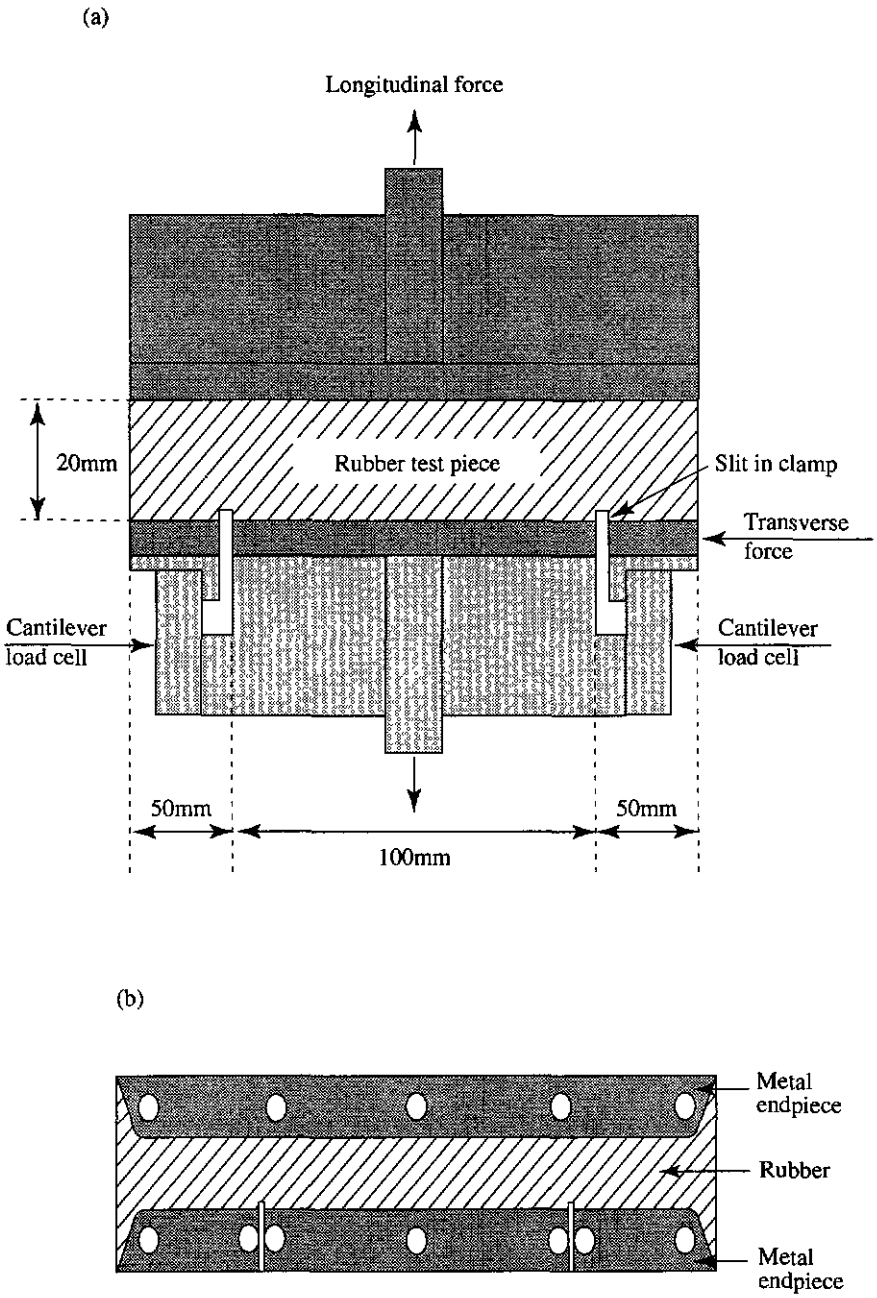



*Figure 2. (a) Split pure shear test rig; (b) bonded pure shear test piece.*

TABLE 1. FORMULATIONS OF MATERIALS USED

| EDS number | 19 | 14 | 15 | 16 |
|---|---|---|---|---|
| Natural rubber (SMR CV60) | 100 | 100 | 100 | 100 |
| Carbon black HAF (*ASTMN-330*) | 1 | 15 | 30 | 45 |
| Process oil[a] | – | 1.5 | 3 | 4.5 |
| Zinc oxide | 5 | 5 | 5 | 5 |
| Stearic acid | 2 | 2 | 2 | 2 |
| Antioxidant/anti-ozonant[b] | 3 | 3 | 3 | 3 |
| Antiozonant wax | 2 | 2 | 2 | 2 |
| Accelerator, CBS[c] | 0.6 | 0.6 | 0.6 | 0.6 |
| Sulphur | 2.5 | 2.5 | 2.5 | 2.5 |
| Cure (min @ 140°C) | 40 | 38 | 40 | 40 |

[a]low viscosity naphthenic oil

[b]N-(1,3-dimethylbutyl)-N'-phenyl-phenylene-diamine

[c]N-cyclohexylbenzothiazole-2-sulphenamide

## Methods

All stress-strain measurements were carried out in an Instron universal test machine. Unless otherwise stated, the cross-head speeds were 2.5 mm $min^{-1}$ (pure shear), 20 mm $min^{-1}$ (tension) or 5 mm $min^{-1}$ (compression) and only the first stress-strain cycle was recorded; this was taken to as high a strain as possible. Displacement was measured using an auxiliary LVDT for pure shear and compression, but in the case of tension the cross-head displacement output was deemed to be sufficiently accurate. All the test pieces were cured to maximum rheometer torque.

## RESULTS

### Magnitude of the Side Force in Pure Shear Tests

Letting $F_1$ and $F_2$ be the forces in the longitudinal (*Equation 1*) and transverse (*Equation 2*) directions, we find from geometrical considerations that:

$$F_1 = wh_0\sigma_1/\lambda_1$$
$$F_2 = \ell_0 h_0 \sigma_2 \quad \ldots 12$$

where $w$, $h_0$ and $\ell_0$ are the unstrained width, thickness and height, respectively of the test pieces.

Substituting *Equations 8* for $\sigma_1$ and $\sigma_2$ into *Equations 12*, noting that in pure shear $\lambda_2 = 1$, leads to:

$$\frac{F_2}{F_1} = \frac{\ell_0 \lambda \sigma_1}{w \sigma_2} = \frac{\ell_0}{w} \frac{\lambda}{(\lambda^2 + 1)} \left\{ \frac{\partial W/\partial I_1 + \lambda^2 \dfrac{\partial W}{\partial I_2}}{\dfrac{\partial W}{\partial I_1} \quad \dfrac{\partial W}{\partial I_2}} \right\} \quad \ldots 13a$$

$$\rightarrow \frac{\ell_0}{2w} \quad \text{at small strain}$$

where for simplicity the subscript has been omitted from $\lambda_1$.

According to Fukahori and Seki[6], Gregory[11], Yeoh[12] and Davies *et al.*[13] $\partial W/\partial I_2$ is negligible for black-filled rubber in comparison to $\partial W/\partial I_1$, so that the term in braces in *Equation 13a* is close to unity:

$$F_2/F_1 \approx \frac{l_0}{w} \frac{\lambda}{(\lambda^2 + 1)} \quad \ldots 13b$$

The experimental measurements of $F_2/F_1$ are compared in *Figure 3* with *Equation 13b*. As required by *Equation 13* the experimental points and theoretical lines coincide near $\lambda = 1$, although the experimental scatter of the points in the low strain region is large. This is not surprising since the precision with which the forces and displacement can be measured is proportionately worse when they are small. Nevertheless $F_2/F_1$ was found to be very close to 0.05 in all cases, as predicted by *Equation 13*.

There is good agreement between the plots for the filled rubber, confirming Gregory's conclusion[11] that $\partial W/\partial I_2$ is negligible. A substantial difference is seen in the case of the unfilled compound indicating a larger importance of the $\partial W/\partial I_2$ term. The presence of non-zero $\partial W/\partial I_2$ is apparent in this form of plot, especially at large values of $\lambda$ because $\partial W/\partial I_2$ is multiplied by $\lambda_1^2$ in the numerator but not in the denominator of the term in braces in *Equation 13a*.

From an engineering point of view, we see from *Figure 3* that $F_2/F_1$ is in all cases quite small, so that failure to measure it with a greater precision than even 30% or so (the maximum discrepancy seen in *Figure 3*) is not likely to have practical significance. Much greater precision would, of course, be desirable in measuring the primary stress $F_1$.

## Calculation of $\partial W/\partial I_1$ and $\partial W/\partial I_2$

*Equations 9* were used to calculate $\partial W/\partial I_1$ and $\partial W/\partial I_2$ from the forces $F_1$ and $F_2$, noting that $\lambda_2 = 1$ and making use of *Equations 12*. The results of these calculations are given in *Figure 4*.

In the case of the highly filled rubbers (EDS 15 and EDS 16), $\partial W/\partial I_2$ is very close to zero for $I_1 - 3$ values greater than about 1, in agreement with Gregory's conclusions[11]. As stated earlier, Rivlin and Saunders[4] thought that the use of *Equations 9* is unlikely to be reliable for $I_1 - 3$ less than about 5; similarly the pronounced changes in $\partial W/\partial I_1$ and $\partial W/\partial I_2$ seen in *Figure 3* as $I_1 - 3$ approaches zero are believed to be artefacts.

We may ask the question, if it is difficult to resolve experimentally separate values for $\partial W/\partial I_1$ and $\partial W/\partial I_2$ in the region of small strain,

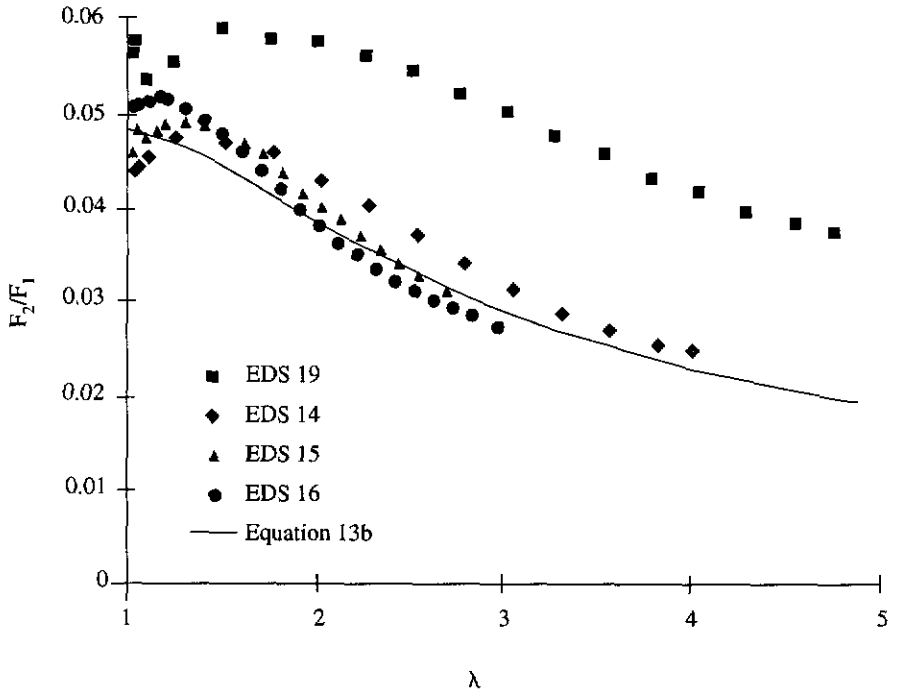



*Figure 3: Plots of ratio of transverse and longitudinal forces $F_2/F_1$ versus extension ratio in pure shear.*

are their individual magnitudes likely to be needed in making engineering calculations? Or is it sufficient to know their sum at small strain?

Rivlin[3] has shown that the secant shear modulus is given by:

$$G = 2\left(\frac{\partial W}{\partial I_1} + \frac{\partial W}{\partial I_2}\right) \qquad \ldots 14$$

Thus twice the sum of the two lines given in *Figure 4* is simply the shear modulus. For filled rubbers, an increase in either $\partial W/\partial I_1$ or $\partial W/\partial I_2$ would be expected with decreasing strain at low strains, corresponding to the high initial modulus[11,13]. This is observed although for EDS 16 most of the increase appears to be attributed to the $\partial W/\partial I_2$ term, whereas for EDS 15, most of the upturn appears in the $\partial W/\partial I_1$ term. From the form of the equations not only are the errors at very low strains likely to be very large, but the error in $\partial W/\partial I_1$ will always be in the opposite direction to the error in $\partial W/\partial I_2$. Fukahori and Seki[6] reported that, at very small values of strain $\partial W/\partial I_1$ was large and positive and decreased rapidly with increasing strain, and $\partial W/\partial I_2$ was large and negative and increased rapidly with increasing strain. Although it is necessary for the sum of the two to increase as the strain approaches

zero (in order to account for the high initial stiffness of filled rubber) it seems likely that much of the apparent effect is attributable to the difficulty of separating $\partial W/\partial I_1$ and $\partial W/\partial I_2$ at low strains. The more random nature of the behaviour at small strains in this work would support this suggestion.

At moderate to high strains $\partial W/\partial I_1$ increases with strain for filled rubber. This is to be expected as the modulus increases, the point of inflection occurring at a lower strain for a higher filler content.

If we make Gregory's assumption[11] at the onset, that $\partial W/\partial I_2 = 0$, then $\partial W/\partial I_1$ may be found directly from just one of the stresses in *Equation 8*. Here, $\sigma_1$ has been chosen, and the resulting plots of $(\partial W/\partial I_1)_{greg}$ are included in *Figure 4*.

## Calculation of $\sigma(\lambda)$, Assuming $W$ is Separable

For pure shear, $\lambda_2 = 1$, $\lambda_3 = 1/\lambda$ and $\sigma_3 = 0$. It follows from *Equation 11b* that:

$$\sigma_1 - \sigma_2 = \phi(\lambda)$$
$$\text{while } \sigma_2 - \sigma_3 = \sigma_2 = \phi(1/\lambda)$$

Hence, from an experiment in pure shear, for which all the principal stresses are measured, the values of the function $\phi(\lambda)$ can be calculated over a range of $\lambda$ from $1/\lambda_{max}$ to $\lambda_{max}$ where $\lambda_{max}$ is the maximum value of the largest extension ratio achieved in the experiment.

Plots of $\phi(\lambda)$ for the four vulcanisates EDS 19, 14, 15 and 16 are given in *Figure 5*.

## Comparison of Predictions of Derived Strain Energy Functions with Experimental Tension and Compression Results

Regarding the split pure shear test piece as a method of determining $W$, it is interesting to see how accurately the results predict the stress-strain behaviour in compression and tension. In terms of the $I_2$ value, for a given $I_1$ value, these deformations are as different as possible from shear, in opposite directions (see *Figure 1*) and therefore provide quite an exacting test.

Three different strain energy functions were used, all based on the pure shear data:

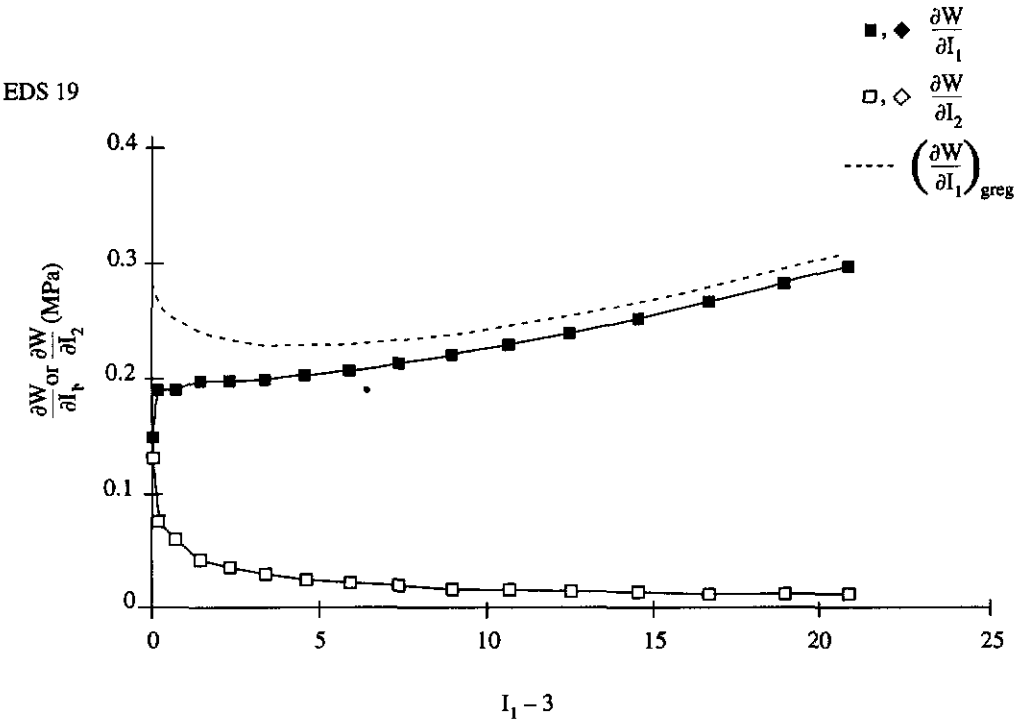
(i) *Rivlin*. This is the most general form, $W$ being regarded as a function of both $I_1$ and $I_2$. However, in order to substitute values of $\partial W/\partial I_1$ and $\partial W/\partial I_2$ from *Figure 4* into *Equation 8* to calculate the compression or tension stress-strain behaviour, assumptions must be made about the dependence of $\partial W/\partial I_1$ and $\partial W/\partial I_2$ on $I_1$ and $I_2$. These assumptions must be compatible with:

$$\frac{\partial}{\partial I_1}\left(\frac{\partial W}{\partial I_2}\right) = \frac{\partial}{\partial I_2}\left(\frac{\partial W}{\partial I_1}\right) \quad \ldots 15$$

The assumption made here is that $\partial W/\partial I_1$ is independent of $I_2$ and $\partial W/\partial I_2$ is independent of $I_1$.

(ii) *Gregory approach*[11]. Using the Rivlin approach but assuming with Gregory at the onset that $\partial W/\partial I_2 = 0$ means that $\partial W/\partial I_1$ is then a function of $I_1$ only, according to *Equation 15* above. *Equations 8* may be used to predict the compression and tension stress-strain behaviour from the plots of $(\partial W/\partial I_1)_{greg}$ given in *Figure 4*.

(a) EDS 19




(b) EDS 14

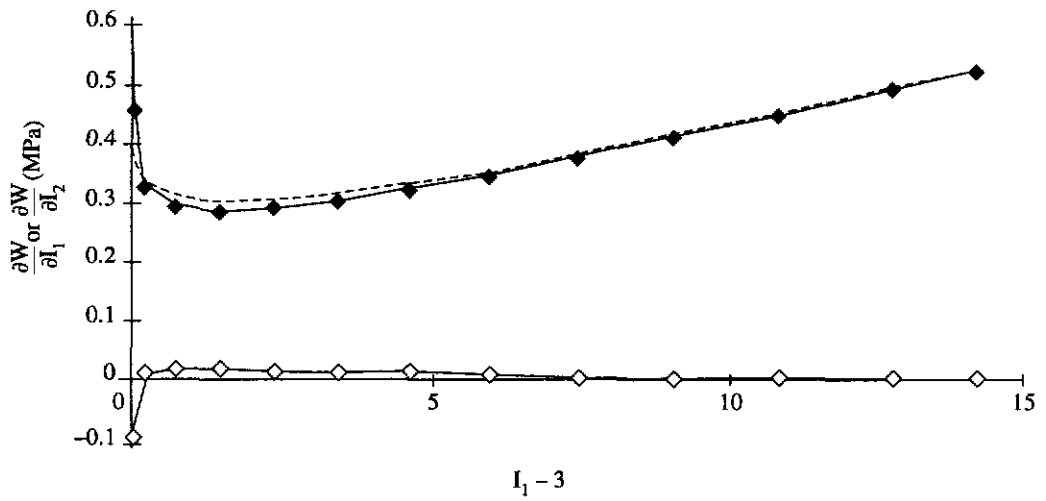
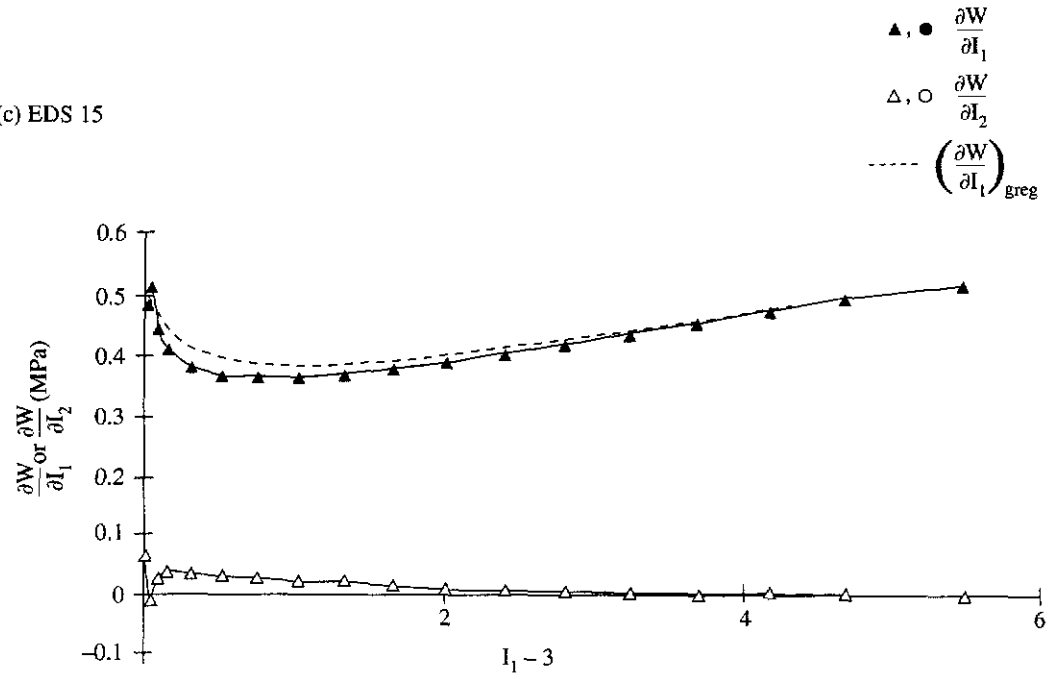



*Figure 4(a) and (b). Plots of values of* $\partial W/\partial I_1$ *and* $\partial W/\partial I_2$ *calculated from* Equations 8 *using the split pure shear test results* versus $I_1 - 3$ *(equal to* $I_2 - 3$*).*

(c) EDS 15




(d) EDS 16

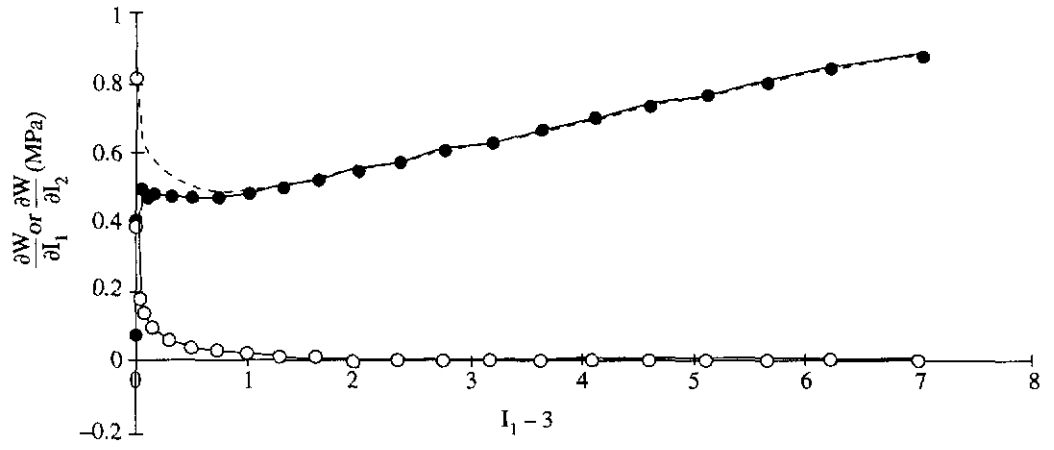



*Figure 4(c) and (d). Plots of values of* $\partial W/\partial I_1$ *and* $\partial W/\partial I_2$ *calculated from* Equations 8 *using the split pure shear test results* versus $I_1 - 3$ *(equal to* $I_2 - 3$*).*

(iii) *Separable strain-energy function approach* The compression and tension stress-strain behaviour may be predicted from *Equation 11b* using the values of $\phi(\lambda)$ given in *Figure 5*.

The predictions of the three approaches are compared to the experimental results in *Figure 6* In all cases the agreement between the three strain energy functions and the experimental results is good This suggests that, for engineering purposes, it is sufficient to choose the simplest of the three assumed forms for $W$ — the Gregory form. It follows that the split shear test piece may be an unnecessarily sophisticated technique for determining $W$; with the Gregory assumption any homogeneous deformation, with measurement of one of the principal stresses, suffices to determine $W$ through *Equation 8*.

## STRESS RELAXATION EXPERIMENTS

The time dependent behaviour of rubber under static or variable strains is of considerable interest Kawabata[14] found that, for unfilled peroxide vulcanisates, under various constant biaxial strains, $\partial W/\partial I_1$ and $\partial W/\partial I_2$ decayed at the same rate. In other words, the strain energy function at any time $t$ may be written in the separable form:

$$W_t = W_t(I_1 I_2) = f(t)\ W(I_1, I_2)_{t=t_o} \qquad 16$$

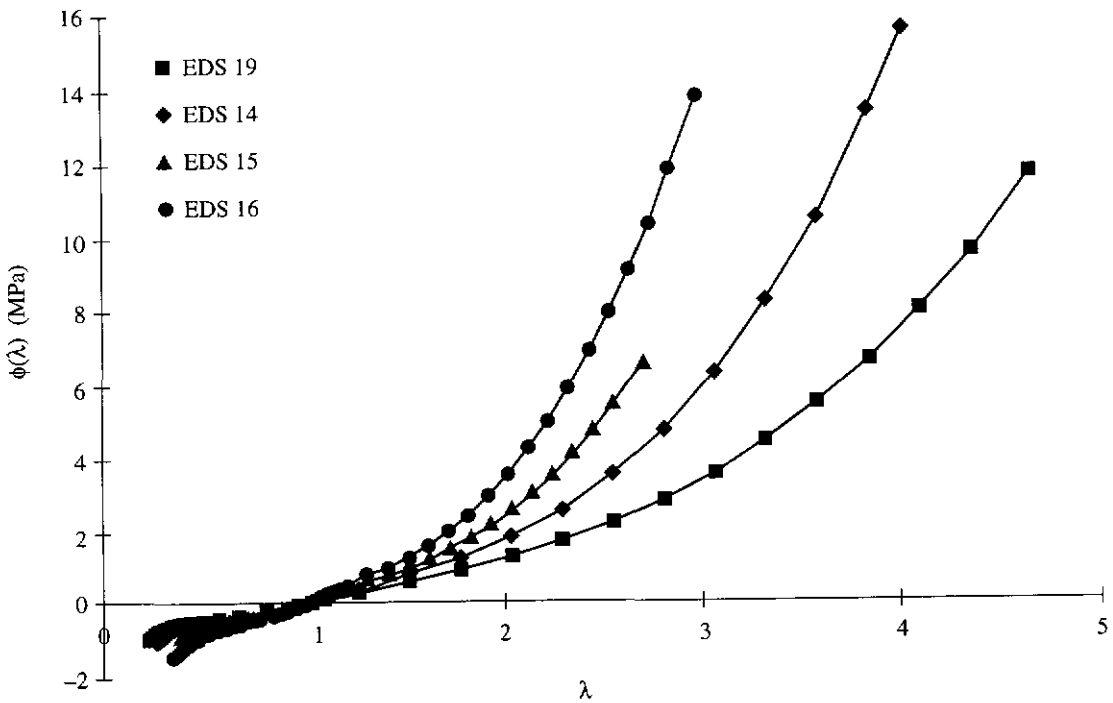



*Figure 5. Plots of the* $\phi(\lambda)$ *(see* Equation 11b*) calculated from the split pure shear test results* versus $\lambda$.

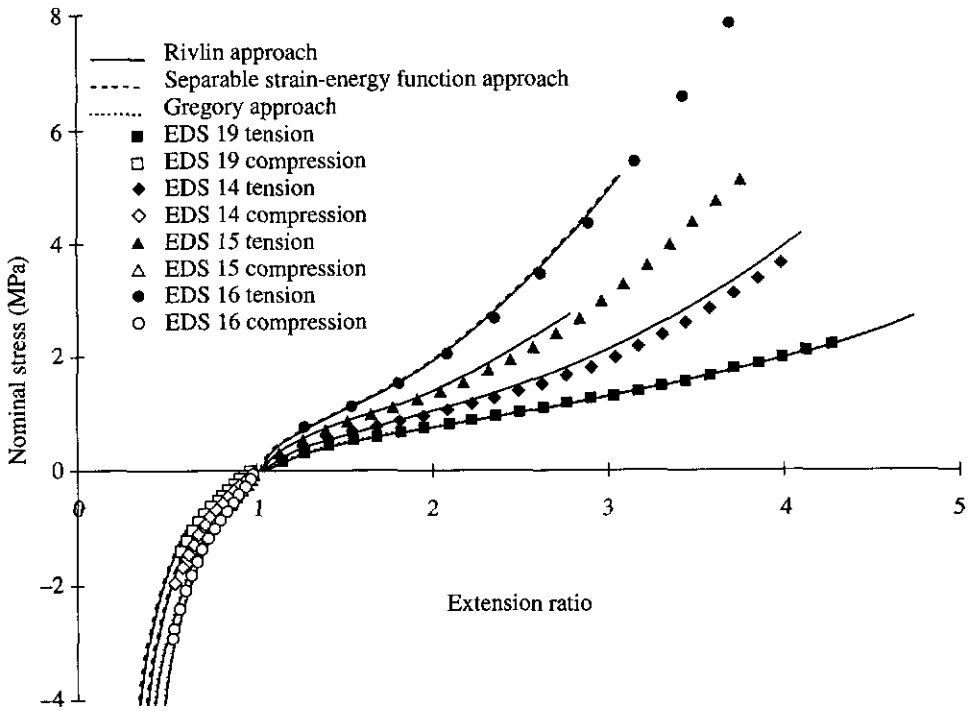



*Figure 6. Comparison of predictions from the split pure shear data of stress-strain behaviour in compression and tension with direct experiment.*

Davies *et al.*[15] have examined the stress relaxation in simple extension under both static and cyclic straining and noted that the relaxation rate increases with increasing strain and filler loading.

The split pure shear experiment provides a relatively straightforward method of examining time dependent behaviour, having the special advantage of enabling simultaneous measurement of two principal stresses without the intervention of friction.

## Stress Relaxation under a Static Strain

In these experiments the split shear test piece was strained to a predetermined extent in 10 sec, and then held constant. The decay of the axial and transverse forces ($F_1$ and $F_2$, respectively) was monitored, and plots made against the logarithm of time. The plots, shown in *Figure 7*, indicate that the rate of relaxation of the transverse force tends to be somewhat lower than for the longitudinal force, whether the vulcanisate is filled or unfilled, casting doubt on the validity of *Equation 16* for the case of sulphur-cured vulcanisates. As expected[15] the rates of stress relaxation are much higher for EDS 16 (NR + 45 p.p.h.r. N330) than for EDS 19 (unfilled NR).

## Cyclic Stress Relaxation Experiments

All measurements reported above have been carried out during the first cycle of a previously

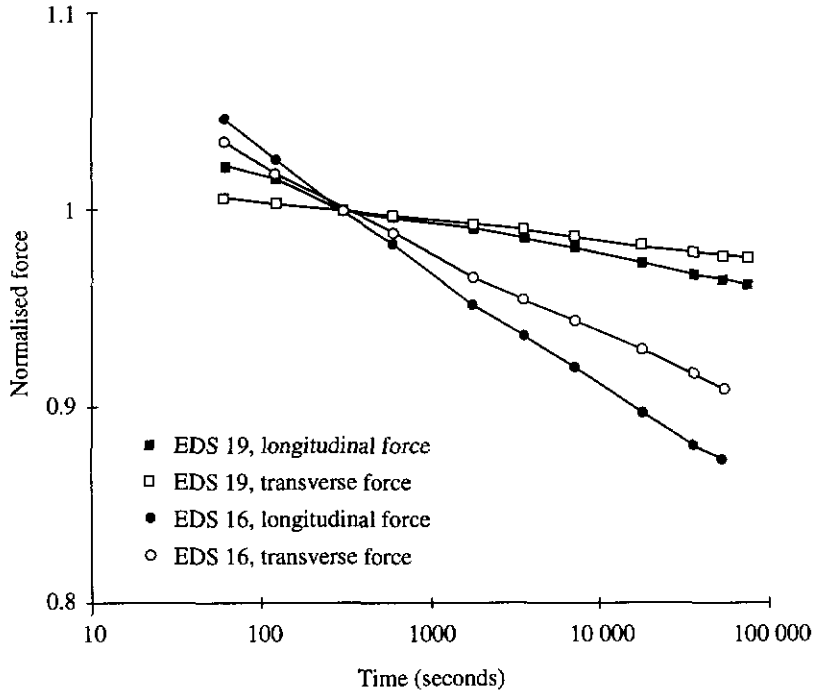



*Figure 7. Comparison of fixed strain stress relaxation for longitudinal and transverse forces in pure shear.*

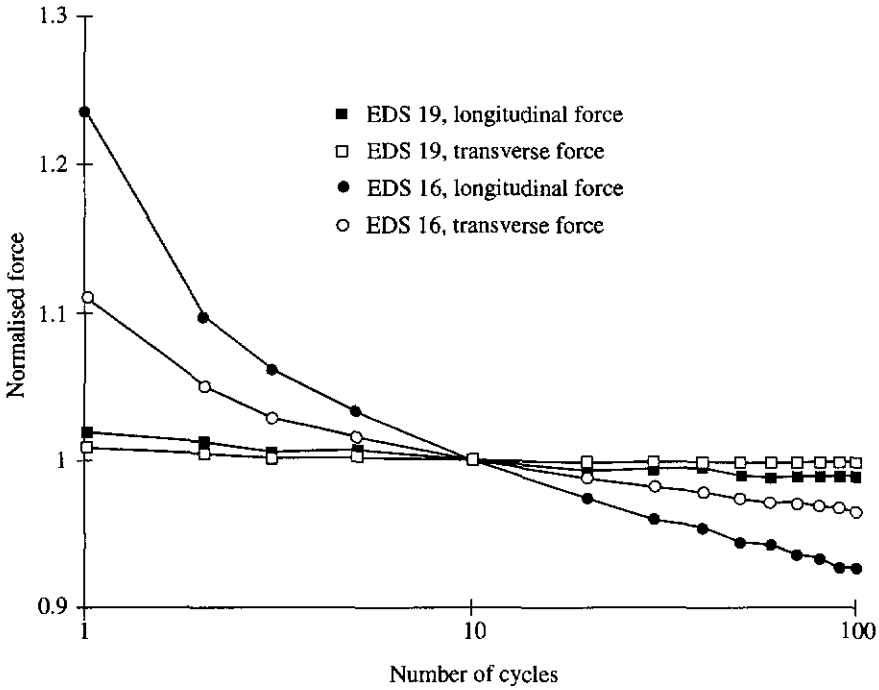



*Figure 8. Comparison of cyclic peak stress relaxation for longitudinal and transverse forces in pure shear.*

undeformed test piece If the test piece is cycled again to the same displacement the peak force reached is lower *Figure 8* shows that this reduction of peak force continues with further cycling, on an approximately linear basis (after the first cycle) with the logarithm of the number of cycles The effect is much larger for the filled vulcanisate, EDS 16 The use of the split shear test piece enables the decay of the peak transverse force to be monitored as well, the decay rate is apparently smaller

## DISCUSSION AND CONCLUSIONS

The technique of using a split pure shear test piece to provide a limited form of biaxial data has enabled an almost general characterisation of filled and unfilled rubber without the need for a fully biaxial rig, with its associated problems of experimental complexity An additional advantage is that the problem of friction in the guide rails of biaxial rigs[16] is absent, which is not the case in alternative simplified rigs such as the 'strip biaxial' rig of Fukahori *et al*[6]

The results obtained for the first strain cycle of sulphur cured natural rubbers confirm that $\partial W/\partial I_2$ is small for unfilled rubber, as found also in the original work of Rivlin and Saunders[4], and negligible for filled rubbers, in agreement with several earlier studies[6,11–13] In any case, it is not possible to resolve $\partial W/\partial I_1$ and $\partial W/\partial I_2$ for small strains, although their sum is determined with reasonable reliability and is sufficient to predict the stresses associated with small strains

Thus for most practical purposes it is recommended that an assumption is made (following Gregory[11]) at the outset that $\partial W/\partial I_2$ is zero, in which case the strain energy function can be determined using stress-strain data from a single uniaxial test The results could also be fitted sufficiently well for practical purposes using the Valanis-Landel assumption of a separable strain energy function but this approach has two disadvantages compared to the Gregory assumptions

(i) A general separable function cannot be determined using a uniaxial test

(ii) The sub-routes for user-defined strain energy functions in most commercial FEA packages call for $W$ to be expressed as a function of $I_1$ and $I_2$, although it is possible to re-express a separable function in this way[17] it is an extra inconvenience

The split pure shear test piece is also suitable for making time-dependent or cyclic stress relaxation measurements, for which its capability of measuring both stresses with no problem of friction is particularly interesting Preliminary results suggest that the time-dependence of $W$ is not separable for sulphur-cured natural rubber vulcanisates, since the transverse force relaxes more slowly than the primary force A similar difference in rate is seen for cyclic stress-relaxation

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