# Some Considerations in Deciding the Optimum Number of Recorded Trees in a Plot for Experiments on Immature Hevea brasiliensis 

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#### Abstract

A description is given of an attempt to determine the optimum number of recorded trees per plot for experiments on young rubber. Plot variation is composed of genetic and environmental variations, but the relative importance of these two factors could not be studied owing to lack of data from uniformity trials. Three cases are therefore discussed. In Case I where the environmental variation is assumed to be negligible, the optimum number of recorded trees in a plot varies according to the accuracy of the plot mean. About 35-40 recorded trees per plot would permit estimation of the plot mean within $\pm 6 \%$ of the mean for girth and within $\pm 12 \%$ of the mean for girth increment (up to about 6 to 7 years from planting). In Case 2, environmental variation is assumed to be independent of plot size and in Case 3, genetic variation is assumed to be negligible. In none of the three cases, is a consistent estimate obtained of the number of recorded trees per plot for which the variance of a treatment mean is at a minimum. The importance of 'guard' zones to protect the recorded trees from the poaching effect of neighbouring plots is stressed.


The optimum number of trees which should be recorded per plot when measuring girth and girth increments is of some importance in fixing an adequate plot size for manuring and other experiments on immature rubber. When the optimum number of trees to be recorded is known, the plot size for a fixed planting distance can be determined by allowing for a suitable number of boundary trees to takeinto account possible poaching effects between different experimental treatments. The number of boundary rows on either side and number of trees at either end of the recorded rows to be provided as 'guard' zones depends on the planting distance, nature and duration of the experiment. It has been shown (Watson and Narayanan, 1963) that for normal planting distances of $30 \times 8 \mathrm{ft}, 24 \times 8 \mathrm{ft}$ and $22 \times 11$ ft , it is sufficient if one boundary row on either side and two trees at either end of the recorded rows are kept as 'guard' zones to protect the experimental treatments from poaching, when the age of trees from planting
is 5-6 years. Larger boundary zones may have to be provided as the trees grow older and their rooting develops (Haines, 1942).

The variability in girth or girth increment of uniformly treated trees is influenced by: (i) genetic factors and (ii) environmental factors. As pointed out by Pearce (1955), the relationship connecting $V_{n}$, the variance per unit area between plots of $n$ trees, and $V_{1}$, the variance between individual trees, is $V_{n}=\frac{V_{1}}{n}$ where $V_{1}$ is independent of the characteristics of the field. The environmental differences also play a part, and from Fairfieldi Smith (1938), this is known to be $\frac{V_{2}}{n^{b}}$, where $V_{2}$ is the variance between single trees due to a position and $b$ is a constant between 0 and $1 . \quad V_{1}$ and $V_{2}$ thus correspond to the genetic and environmental components of the total variation. If these two sources of variation are assumed to be independent and additive, then it follows that

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$$
\begin{equation*}
V_{n}=\frac{V_{1}}{n}+\frac{V_{2}}{n^{b}} \tag{1}
\end{equation*}
$$

Similarly, the tree variation within plots ( $V$ ) is given (Shrikhande, 1957) by,

$$
\begin{equation*}
V=V_{1}+\frac{n}{n-1}\left(1-n^{-b}\right) V_{2} \tag{2}
\end{equation*}
$$

The relative importance of these two sources of variation could be studied by means of uniformity trials. Because data from uniformity trials have not been readily available, recourse is made to data from one of a number of large scale cover plant experiments on immature rubber which have been in existence since 1957.

The details of the experiment and data used for this study are listed in Table 1. Regular girth measurements have been kept since March 1959 (i.e. 18 months from planting) for all the recorded trees in each experimental plot. The following three cases are considered:

1. Environmental variation negligible.
2. Environmental variation independent of plot size.
3. Genetic variation negligible.

## CASE 1. ENVIRONMENTAL VARIATION NEGLIGIBLE

If the environmental variation is negligible and can be ignored, then plot size has little effect upon the accuracy of the experiment, provided there are no 'guard' zones and the experimental area is kept constant. But for experiments on Hevea, 'guard' zones are a pre-requisite for experimental plots and hence it is important to know how many recorded trees constitute an optimum number in a plot and how this is to be determined. The number of recorded trees in a plot is said to be optimum if the mean value of either girth or girth increment based on the optimum number of trees does not

TABLE 1. DETAILS OF THE EXPERIMENT AND DATA UTILISED

## Experimental details

## Site

Date of planting Planting material Planting distance Original stand per acre Plot size
No. of trees recorded Design

Covers
Fertiliser treatments Methods of application

Sepang Estate
21 Sept. to 2 Oct. 1957
PBIG/GGI stumped seedlings
$30 \times 8 \mathrm{ft}$
181
5 rows of 22 trees $=110$ trees, or 0.61 acre
3 rows $\times 18$ trees $=54$
Split-plot: Covers in main plots and fertiliser treatments in sub-plots
Legumes, Grass, Mikania and Naturals
Fertiliser mixture at 0 and 1 levels
To clean weeded planting rows (r) or to the covers (c)

Data utilised

| Girth | Girth Increment |  |  |
| :---: | :---: | :---: | :---: |
| 1. Sept. 1959 | 7 | March to Sept. 1960 | (6 months) |
| 2. Sept. 1960 | 8. | Sept. 1959 to March 1960 | (6 months) |
| 3. Aug. 1961 |  | March to Sept. 1960 | (6 months) |
| 4. Aug. 1962 |  | Sept. 1960 to Aug. 1961 |  |
| 5. Oct. 1963 | 11. | Aug. 1961 to Aug. 1962 |  |
| 6. April 1964 | 12. | Aug. 1962 to Oct. 1963 | (1 year and 3 months) |

differ from the true value by more than a given percentage of the mean.

For each of the 48 plots and for each of the periods mentioned above, the means, standard deviations (s.d.) and hence coefficients of variation (c.v. $=100 \mathrm{~s} . \mathrm{d} . /$ mean) have been calculated. The number of recorded trees varies from plot to plot owing to vacancies and the number of recorded trees in a plot has fallen with time because of losses due to disease or wind damage. The means of the different plots and their corresponding coefficients of variation show no relationship for any of the periods, thereby suggesting that the c.v.'s do not depend on the means of the different experimental treatment plots. Figure 1 shows the absence of any relationship between the mean girth per plot and the coefficient of variation of the different plots for September, 1959. Thus the c.v.'s of the different plots for any one period can be considered as a random sample from a population of c.v.'s. The coefficients of variation of the means (c.v.m.) of the different plots have also been worked out by dividing the different c.v.'s by the square root


Figure 1. Relationship between mean girth per plot and the corresponding coefficient of variation.
of the number of observations on which the means are based.

Frequency distributions of the c.v.'s and also of the c.v.m.'s of the girth and girth increment data have been made for each of the periods. The c.v. and also the c.v.m. variation for girth data is smaller for September, 1959 (varying between 9 to $19 \%$ and 1.3 to $2.7 \%$ respectively) than in later years when the c.v.'s and hence c.v.m.'s varied respectively, between 11 to $23 \%$ and 1.7 to $3.5 \%$. For girth increments, out of the first 3 half-yearly periods, the first period shows the smallest c.v. variation, between 16 to $36 \%$ (c.v.m. varying between 2.2 to $5.2 \%$; the second half-yearly period shows somewhat larger variation in c.v. and also of c.v.m. For the third half-yearly period, the variation in c.v. is somewhat similar to the variation obtaining for c.v. for the three yearly increments (c.v. varying between 16 to $48 \%$ and c.v.m. varying from 2.5 to $7.3 \%$ ). For the sake of simplicity, only the frequency distributions of girths and girth increments for c.v.m. for each of the periods have been shown in Tables 2 and 3. The distribution of c.v. or c.v.m. is symmetrical for girths but slightly skew for girth increments.

The distribution of the tree-to-tree girth measurements in any plot follows approximately a symmetrical law for any of the periods and the distribution is somewhat asymmetrical for girth increments. If $\bar{x}$ is the observed plot mean (either girth or girth increment) based on $n$ trees, the distribution of $\bar{x}$ follows the normal law, even if the original distribution is somewhat skew or asymmetrical.

By normal law, we have

$$
\begin{align*}
& \quad \mathbf{P}\left(|\bar{x}-a| \leqslant k\left(\frac{\sigma}{\sqrt{n}}\right)\right)=a \\
& \text { i.e. } \quad \mathbf{P}\left(|\bar{x}-a| \leqslant k\left(\frac{\sigma}{\bar{x} \sqrt{n}}\right) \bar{x}\right)=\alpha \\
& \text { i.e. } \quad \mathbf{P}(|\bar{x}-a| \leqslant k(\text { c.v.m. }) \bar{x})=a
\end{align*}
$$

where P denotes the probability $a$ is the confidence coefficient $\sigma$ is the standard deviation $a$ is the true plot mean
and $\quad k$ is a constant, depending on $a$

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TABLE 2. FREQUENCY DISTRIBUTION OF C.V.M. FOR GIRTH MEASUREMENTS

| Class interval | Frequency |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Sept. '59 | Sept. '60 | Aug. '61 | Aug. '62 | Oct. '63 | April '64 |
| 1.3-1.4 | 1 |  |  |  |  |  |
| 1.4-1.5 | - |  |  |  |  |  |
| 1.5-1.6 | 3 |  |  |  |  |  |
| $1.6-1.7$ | 2 |  |  |  |  |  |
| 1.7-1.8 | 5 |  |  |  |  | 1 |
| 1.8-1.9 | 5 |  |  |  | 3 | 1 |
| 1.9 <br> 2.0 <br> - 2.0 <br> 2.1 | $\stackrel{9}{5}$ | 2 |  | - | 3 | 1 |
| $2.1-2.2$ | 6 | 2 | 2 | 4 | 5 | 6 |
| $2.2-2.3$ | 2 | 7 | - | 2 | 6 | 5 |
| 2.3-2.4 | 5 | 8 | 3 | 6 | 3 | 6 |
| $2.4-2.5$ | 2 | 3 | 1 | 5 | 7 | 3 |
| 2.5-2.6 | 2 | 7 | 5 | 5 | 6 | 6 |
| $2.6-2.7$ | 1 | 7 | 8 | 8 | 6 | 3 |
| $2.7-2.8$ |  | 2 | 9 | 5 | 4 | 7 |
| 2.8-2.9 |  | 3 | 5 | 3 | 2 | 4 |
| 2.9-3.0 |  | 3 | 4 | 3 | $\overline{1}$ | 2 |
| 3.0-3.1 |  | 1 | 3 | 2 | 1 | 1 |
| 3.1 3.2 3.3 .2 3.3 |  | 3 | 3 3 | $\stackrel{1}{2}$ | 1 | 1 |
| $3.3-3.4$ |  |  |  | 1 | - | - |
| 3.4-3.5 |  |  | 1 |  |  |  |
| Total | 48 | 48 | 48 | 48 | 48 | 48 |

TABLE 3. FREQUENCY DISTRIBUTION OF C.V.M. FOR GIRTH INCREMENTS

| Class interval | Frequency |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \text { March to } \\ & \text { Sept. } 59 \end{aligned}$ | $\begin{aligned} & \text { Sept. '59 } \\ & \text { March '60 } \end{aligned}$ | March to Sept. '60 | Sept. '60 <br> to Sept. '61 | Sept. '61 to Aug. '62 | Aug. '62 to Oct. ' 63 |
| 2.2-2.5 | 1 |  |  |  |  |  |
| 2.5-2.8 | 5 |  |  |  |  | 1 |
| $2.8-3.1$ | 9 |  |  |  |  | 1 |
| $3.1-3.4$ | 12 |  |  |  |  | $\overline{7}$ |
| $3.4-3.7$ $3.7-4.0$ | 10 4 |  |  | ${ }_{8}^{1}$ | 2 | 7 |
| 4.0-4.3 | 4 |  | 6 | 9 | 6 | 13 |
| $4.3-4.6$ | 2 | 1 | 6 | 4 | 6 | 4 |
| $4.6-4.9$ | - | 3 | 11 | 10 | 9 | 3 |
| $4.9-5.2$ | 1 | 8 | 8 | 6 | 8 | 8 |
| $5.2-5.5$ |  | 5 | 8 | 3 | 3 | 1 |
| $5.5-5.8$ |  | 12 | 2 | 3 | 2 | 2 |
| 5.8-6.1 |  | 5 | 3 | 1 | 1 | 1 |
| $6.1-6.4$ |  | 5 | 1 | - | 1 |  |
| $6.4-6.7$ |  | 2 |  | 1 | 1 |  |
| $6.7-7.0$ |  | 2 |  | 1 | $\overline{2}$ |  |
| $7.0-7.3$ |  | 2 |  |  | 2 |  |
| $7.3-7.6$ 7.6 |  | 1 |  |  |  |  |
| Total | 48 | 48 | 48 | 48 | 48 | 48 |

Under normality, $k=2$ when $a=0.95$. Thus we notice the accuracy of $|\bar{x}-a|$ mainly depends on the observed coefficient of variation of the mean. By increasing the sample size, it is possible to reduce the c.v.m. to any desired level and thus increase the accuracy. It is known that c.v.m. varies inversely as the square root of the number on which the mean is based, i.e.

$$
\text { c.v.m. } 1 \propto \frac{1}{\sqrt{ } n_{1}} \text { (based on } n_{1} \text { obser- }
$$

Similarly c.v.m. $2 \propto \frac{1}{\sqrt{n_{2}}}$ (based on $n_{2}$ obser-
Taking the ratio,

$$
\begin{gather*}
\frac{\text { c.v.m.1 }}{\text { c.v.m.2 }}=\frac{\sqrt{ } n_{2}}{\sqrt{ } n_{1}} \\
\sqrt{ } n_{2}=\frac{(\text { c.v.m.1 }) \sqrt{ } n_{1}}{(\text { c.v.m.2 })} \\
n_{2}=\frac{(\text { c.v.m.1 })^{2} n_{1}}{(\text { c.v.m.2 })^{2}}=\frac{(\text { c.v. })^{2}}{(\text { c.v.m.2 })^{2}} \tag{4}
\end{gather*}
$$

Thus, if c.v.m. 1 is the observed coefficient of variation of the mean based on $n_{1}$ observations, and if it is desired to bring it down to an allowable or desired coefficient of variation of the mean, say, c.v.m.2, then the new sample size is given by (4).

It is usually the practice to reduce the maximum observed coefficient of variation of the mean to a desired level and then obtain the sample size required. For each of the periods considered, we have 48 values of c.v.m.'s and if the maximum value is chosen, undesirably large sample numbers will be required. As it is not possible to ascertain the chance of the maximum occurring in this or future repetitions of the experiments, the alternative method of considering the frequency distributions of the observed c.v.m.'s (or c.v.'s) for each of the periods has been chosen; the c.v.m.'s (or c.v.'s) that will not be exceeded in $50 \%, 90 \%$ and $95 \%$ of the cases are worked out graphically from the percentage cumulative distributions. To avoid confusion, the per-
centage cumulative distributions of c.v. for girths and girth increments only are shown in Figures 2 and 3. Strictly, one should fit mathematically the appropriate theoretical distributions to the observed models, and then work out the values for the different percentages as indicated above. It is to be noted that the $50 \%$ point from the graphs will not be much different from the $50 \%$ point worked out from the theoretical distributions, while the $90 \%$ and $95 \%$ points can be read only approximately from the graphs since these are end points. The fitting of mathematical equations to the observed distributions has not been attempted, partly because the fitting of mathematical models to these observed distributions is laborious and partly because the object is to show how the method is used rather than to demonstrate its accuracy.

Table 4 shows the different percentage points namely $50 \%, 90 \%$ and $95 \%$ as read from the percentage cumulative graphs for each of the periods for both c.v. and c.v.m. Since the number of observations varies between the different plots for any one period, the c.v. values rather than c.v.m. values have been used for deriving the optimum sample size. The desired levels of c.v.m.'s have been put as $2.5,3,4$ and $5 \%$ for girths, whereas for girth increments, the desired levels have been kept at $5,6,8$ and $10 \%$. The optimum numbers of trees have been worked out using equation (4) and these are tabulated in Table 5. Using the $50 \%$ points of the c.v. distributions, this table shows that about 15 recorded trees would be sufficient for assessing the plot girth mean for any year (2nd year from planting to 7th year after planting) when the desired level of c.v.m. is $5 \%$ (accuracy $=k \times 5 \%$ of the mean $= \pm 10 \%$ of the mean) while about 15 recorded trees are necessary for plot mean girth increment (either half-yearly or yearly in these periods) for a desired c.v.m. level of $10 \%$. However, if one wants to be more accurate in the estimation of plot mean for either girth or girth increment, more recorded trees will be necessary. About $35-40$ recorded trees would be necessary to estimate the mean girth correct to within $\pm 6 \% \bar{x}$, and girth increment to within $\pm 12 \% \vec{x}$.







Figure 2. Percentage cumulative distributions of coefficient of variation for girth.

Still higher numbers of recorded trees are necessary when $90 \%$ or $95 \%$ c.v. points are considered.

CASE 2. ENVIRONMENTAL VARIATION INDEPENDENT OF PLOT SIZE
For this experiment with about 40 recorded trees in a plot, the plot variation is of the order of $5 \%$ of the mean for girths and about $10 \%$ of the mean for girth increments, for each of


Figure 3. Percentage cumulative distributions of coefficient of variation for girth increment.
the periods considered. The tree variation within plots is accepted to an extent of $20 \%$ of the mean for girths and about $40 \%$ of the mean for girth increments (obtained from the $90 \%$ points of the c.v. distributions-see Table 4). Let us assume now that the component of environmental variation remains unaltered with different numbers of recorded trees in a plot, and hence with different plot sizes. In other words, we are assuming the hetero-

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TABLE 4. PERCENTAGE POINTS OF THE C.V. AND C.V.M. DISTRIBUTIONS

| Periods | c.V. |  |  | C.V.M. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 50\% | 90\% | 95\% | 50\% | 90\% | 95\% |
| Girths |  |  |  |  |  |  |
| September '59 | 14.1 | 16.6 | 17.3 | 2.01 | 2.43 | 2.53 |
| September '60 | 16.4 | 19.5 | 20.3 | 2.50 | 2.95 | 3.05 |
| August '61 | 17.9 | 21.0 | 21.8 | 2.77 | 3.18 | 3.30 |
| August '62 | 16.3 | 19.0 | 19.7 | 2.60 | 3.05 | 3.15 |
| October '63 | 15.5 | 17.7 | 18.3 | 2.44 | 2.84 | 2.97 |
| April '64 | 15.7 | 17.8 | 18.3 | 2.49 | 2.88 | 3.00 |
| Girth increments, |  |  |  |  |  |  |
| March to Sept. '59 | 24.0 | 28.5 | 30.0 | 3.35 | 4.15 | 4.42 |
| Sept. '59 to March '60 | 37.5 | 45.8 | 48.0 | 5.70 | 6.85 | 7.25 |
| March to Sept. '60 | 32.0 | 36.8 | 38.5 | 4.83 | 5.70 | 5.95 |
| Sept. '60 to Aug. ' 61 | 29.3 | 36.5 | 38.8 | 4.53 | 5.70 | 6.00 |
| Aug. '61 to Aug. '62 | 29.3 | 36.3 | 38.8 | 4.68 | 5.80 | 6.20 |
| Aug. '62 to Oct. '63 | 26.0 | 32.3 | 33.9 | 4.30 | 5.30 | 5.60 |

Note: These values have been read from the graphs of the percentage cumulative distributions.

TABLE 5. NUMBER OF RECORDED TREES NECESSARY FOR ESTIMATING THE TRUE PLOT MEAN WITH CONFIDENCE LIMITS NOT GREATER THAN $\bar{x} \pm 2$ (c.v.m.) $\bar{x}$

| Periods | 50\% | 90\% | 95\% | 50\% | 90\% | 95\% | 50\% | 90\% | 95\% | 50\% | 90\% | 95\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Desired levels of c.v.m. for girths |  |  |  |  |  |  |  |  |  |  |  |
|  | $2.5 \%$ |  |  | $3 \%$ |  |  | $4 \%$ |  |  | $5 \%$ |  |  |
| Girths |  |  |  |  |  |  |  |  |  |  |  |  |
| Sept. '59 | 32 | 44 | 48 | 22 | 31 | 33 | 12 | 17 | 19 | 8 | 11 | 12 |
| Sept. '60 | 43 | 61 | 66 | 30 | 42 | 46 | 17 | 24 | 26 | 11 | 15 | 16 |
| Aug. '61 | 51 | 71 | 76 | 36 | 49 | 53 | 20 | 28 | 30 | 13 | 18 | 19 |
| Aug. '62 | 42 | 58 | 62 | 29 | 40 | 43 | 17 | 23 | 24 | 11 | 14 | 15 |
| Oct. '63 | 38 | 50 | 54 | 27 | 35 | 37 | 15 | 20 | 21 | 10 | 13 | 13 |
| April '64 | 39 | 51 | 54 | 27 | 35 | 37 | 15 | 20 | 21 | 10 | 13 | 13 |
|  | Desired levels of c.v.m. for girth increments |  |  |  |  |  |  |  |  |  |  |  |
|  | $5 \%$ |  |  | $6 \%$ |  |  | $8 \%$ |  |  | $10 \%$ |  |  |
| Girth increments, |  |  |  |  |  |  |  |  |  |  |  |  |
| March to Sept. '59 | 23 | 32 | 36 | 16 | 23 | 25 | 9 | 13 | 14 | 6 | 8 | 9 |
| Sept. '59 to March '60 | 56 | 84 | 92 | 39 | 58 | 64 | 22 | 33 | 36 | 14 | 21 | 23 |
| March to Sept. '60 | 41 | 54 | 59 | 28 | 38 | 41 | 16 | 21 | 23 | 10 | 14 | 15 |
| Sept. '60 to Aug. '61 | 34 | 53 | 60 | 24 | 37 | 42 | 13 | 21 | 24 | 9 | 13 | 15 |
| Aug. '61 to Aug. '62 | 34 | 53 | 60 | 24 | 37 | 42 | 13 | 21 | 24 | 9 | 13 | 15 |
| Aug. '62 to Oct. '63 | 27 | 41 | 46 | 19 | 28 | 32 | 11 | 16 | 18 | 7 | 10 | 12 |

Note: For these numbers of recorded trees in a plot, the $95 \%$ confidence interval for the true mean will be no larger than $\bar{x} \pm 2$ (c.v.m.) $\bar{x}$. The percentage points of the c.v.m. show the probability that the c.v.m.'s will not be exceeded in $50 \%, 90 \%$ and $95 \%$ of the cases. These percentage point c.v.m.'s have been reduced to the desired c.v.m. levels and hence the optimum number of recorded trees has been obtained (see formula (4)). $\bar{x}$ is the new sample mean based on these numbers and ' $a$ ' is the true plot mean.
geneity coefficient $b$ to be zero. For $n$ trees in a plot, the plot variation $V_{n}$ is comprised of variation ascribable to environmental variation $V_{2}$ and tree variation within plots $V$. When $b=0$, from equation (2), $V_{1}=V$ and hence from equation (1)

$$
\begin{equation*}
V_{n}=V_{2}+\frac{V}{n} \tag{5}
\end{equation*}
$$

Table 6 shows the variation attributable to trees within plots as affected by the number of recorded trees in a plot. It can be seen that for more than 60 trees, the influence of the number of trees is slight.
For $n=40$, we have
$\left.\begin{array}{rl}V_{n} & =(0.05)^{2} \cdot \bar{x}^{2}=0.0025 \bar{x}^{2} \\ & \text { for girths } \\ & =(0.10)^{2} \cdot \bar{x}^{2}=0.01 \bar{x}^{2} \\ & \text { for girth } \\ & \text { increments }\end{array}\right\}$
Similarly

$$
\left.\begin{array}{rl}
V & =(0.20)^{2} \cdot \bar{x}^{2}=0.04 \bar{x}^{2}  \tag{7}\\
& \text { for girths } \\
& =(0.40)^{2} \cdot \bar{x}^{2}=0.16 \bar{x}^{2}
\end{array} \quad \text { for girth } \quad \text { increments }\right\}
$$

From equation (5)


Solving for $V_{2}$ in these two cases,

$$
\left.\begin{array}{rl}
V_{2} & =0.0015 \bar{x}^{2}  \tag{9}\\
& =0.0036 \bar{x}^{2}
\end{array} \quad \begin{array}{l}
\text { for girths } \\
\\
\\
\text { increments }
\end{array}\right\}
$$

The variance of a treatment mean $\left(V_{t}\right)$ based on $r$ replications

$$
=\frac{V_{2}+\frac{V}{n}}{r}
$$

Thus,
$V_{t}=\frac{\bar{x}^{2}}{r}(0.0015+0.04 / n) \quad$ for girths
Similarly,
$V_{t}=\frac{\bar{x}^{2}}{r}(0.0036+0.16 / n) \quad \begin{aligned} & \text { for girth } \\ & \text { increments }\end{aligned}$
Let us also assume that the recorded trees are protected by one boundary row on either side of the recorded rows and one or two trees
at either end of the recorded rows as illustrated below. For simplicity, only square plots in the recorded area have been considered. For example, for a recorded area of 9 trees, the plot will be diagramatically as in either Figure $4(a)$ or $4(b)$. Procedure $4(b)$ is the one usually adopted for manuring experiments with Hevea when the planting distance is either $24 \times 12 \mathrm{ft}$ or $22 \times 11 \mathrm{ft}$. However, as stated earlier, 'guard' zones to be provided must depend on the planting distance, nature, and duration of the experiment itself, and of course, the recording area need not be a square. For a given planting distance, the number of recorded trees and 'guard' zones, the number of recorded rows and the number of points in a recorded row should be chosen so that the proportion of recorded trees to the total number of trees in a plot is as high as possible.

TABLE 6. EFFECT OF NUMBER OF RECORDED TREES PER PLOT ON TREE VARIATION WITHIN PLOTS

|  | Girths | Girth increments |
| :---: | :---: | :---: |
| No. of recorded trees $=n$ | $\frac{\sqrt{ } V}{\sqrt{ } n}(\sqrt{ } V=20 \% \bar{x})$ <br> (\% of mean) | $\frac{\sqrt{ } V}{\sqrt{n}}(\sqrt{ } V=40 \% \bar{x})$ <br> (\% of mean) |
| 10 | 6.32 | 12.64 |
| 15 | 5.16 | 10.32 |
| 20 | 4.47 | 8.94 |
| 25 | 4.00 | 8.00 |
| 30 | 3.65 | 7.30 |
| 35 | 3.38 | 6.76 |
| 40 | 3.16 | 6.32 |
| 45 | 2.98 | 5.96 |
| 50 | 2.83 | 5.66 |
| 55 | 2.70 | 5.40 |
| 60 | 2.58 | 5.16 |
| 65 | 2.48 | 4.96 |
| 70 75 | 2.39 2.31 | 4.78 4.62 |
|  |  |  |

Tables 7 and 8 give the variance of a treatment mean for girths and girth increments for different numbers of recorded trees in a plot when the number of trees per treatment has been kept constant at 200 for all the three cases considered. The number of replications or plots for any treatment should be an integer, but for comparison in the tables, fractions
have been allowed. It is noticed that for this case, the minimum variance of a treatment mean for girth or girth increment (using formulae (10)) occurs when recorded trees number respectively either 16 or 25 .

(b)

| $G$ | $G$ | $G$ | $G$ | $G$ |
| :--- | :--- | :--- | :--- | :--- |
| $G$ | $G$ | $G$ | $G$ | $G$ |
| $G$ | $A$ | $R$ | $R$ | $G$ |
| $G$ | $R$ | $R$ | $R$ | $G$ |
| $G$ | $R$ | $R$ | $R$ | $G$ |
| $G$ | $G$ | $G$ | $G$ | $G$ |
| $G$ | $G$ | $G$ | $G$ | $G$ |

Figure 4. Guard zones. (a) One boundary row each side of recorded rows; one tree at end of each recorded row. (b) One boundary row each side of recorded rows; two trees at end of each recorded row.

## CASE 3. GENETIC VARIATION NEGLIGIbLE

Let us assume now that the genetic variation is negligible and that the variation between large plots is greater than the variation between small plots because of soil heterogeneity. In other words, let us assume that the formula given by Fairfield Smith (1938) holds good in such cases.

The relationship is expressed as:

$$
\begin{equation*}
V_{n}=\frac{V_{2}}{n^{b}} \tag{11}
\end{equation*}
$$

where $V_{n}=$ variance per unit area between plots of $n$ trees
$V_{2}=$ variance between single trees due to a position
$b=$ index of soil variability, varying between 0 and 1 .
The variance $V_{2}$ between single trees due to a position is not known and as an approximation, the variance $V$ between trees within plots has been substituted (see equation (2)).

For a recorded area of about 40 trees in a plot, the plot variation and tree variation within plots are known ( 5 and $20 \%$ of the
mean girths and 10 and $40 \%$ of the mean girth increments respectively):

$$
\left.\begin{array}{r}
40^{b=(0.20)^{2}} \cdot \bar{x}^{2} /(0.05)^{2} \cdot \bar{x}^{2}=16 \text { for girths } \\
40^{b}=(0.40)^{2} \cdot \bar{x}^{2} /(0.10)^{2} \cdot \bar{x}^{2}=16 \text { for girth }  \tag{12}\\
\text { increments }
\end{array}\right\}
$$

$b$ is calculated as 0.75 for both girth and girth increment. The variance of a treatment mean ( $V_{t}$ ) based on $r$ replications

$$
=\frac{V}{r . n^{0.75}}
$$

Thus, $V_{t}=\frac{0.04 \bar{x}^{2}}{r . n^{0.75}} \quad$ for girths

$$
=\frac{0.16 \bar{x}^{2}}{r \cdot n^{0.75}} \quad \begin{align*}
& \text { for girth }  \tag{13}\\
& \text { increments }
\end{align*}
$$

The variances of treatment means under this approach for different numbers of recorded trees have been calculated, and are tabulated also in Tables 7 and 8. It can be seen that the minimum variance of a treatment mean for girth and girth increment occurs respectively when the recorded trees are 36 and 64.

One can easily see that Case 1 is similar to Case 3 when $b=1$ and as such the variances of treatment means for Case 1 have also been tabulated in Tables 7 and 8, for comparison with the other two cases. The minimum variance of a treatment mean for girth and girth increment is obtained when the number of recorded trees is 100 each.

## DISCUSSION

It is evident, from the above three cases, that there is no unique value of the number of recorded trees in a plot for which the variance of a treatment mean is minimum. However, from Tables 7 and 8 , one can infer that approximate minimum variance is obtained when the number of recorded trees in a plot is in the region of 25 to 49 . The minimum value thus depends on the assumptions under which the variance of a treatment mean is derived (i.e. the adequacy of 'guard' zones, the size and shape of plot, the variations of trees within plots and between plots, and also the heterogeneity coefficient).

## TABLE 7. VARIANCES OF TREATMENT MEANS BASED ON DIFFERENT NUMBERS OF RECORDED TREES IN A PLOT

(Minimum values in bold type. The 'guard' zone is given by one boundary row on either side of the recorded rows and one tree at either end of the recorded rows).

| No. of recorded trees $=\boldsymbol{n}$ | Number of trees in a plot $=N$ | No. of replications$r=\frac{200}{N}$ | $\frac{\text { Variance }}{\bar{x}^{2}}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Girth | $\text { se } 1$ <br> Girth increment | Girth | se 2 Girth increment | Girth | 3 <br> Girth increment |
| 1 | 9 | 22.22 | 0.001,800 | 0.007,200 | 0.001,868 | 0.007,363 | 0.001,800 | 0.007,201 |
| 4 | 16 | 12.50 | 0.000,800 | 0.003,200 | 0.000,920 | 0.003,488 | 0.001,131 | 0.004,525 |
| 9 | 25 | 8.00 | 0.000,556 | 0.002,224 | 0.000,738 | 0.002,675 | 0.000,962 | 0.003,848 |
| 16 | 36 | 5.56 | 0.000,450 | 0.001,800 | 0.000,719 | 0.002,446 | 0.000,899 | 0.003,597 |
| 25 | 49 | 4.08 | 0.000,392 | 0.001,568 | 0.000,760 | 0.002,451 | 0.000,877 | 0.003,508 |
| 36 | 64 | 3.13 | 0.000,355 | 0.001,420 | 0.000,831 | 0.002,556 | 0.000,870 | 0.003,479 |
| 49 | 81 | 2.47 | 0.000,330 | 0.001,320 | 0.000,931 | 0.002,794 | 0.000,874 | 0.003,497 |
| 64 | 100 | 2.00 | 0.000,312 | 0.001,248 | $0.001,050$ | 0.003,050 | 0.000,884 | 0.003,535 |
| 81 | 121 | 1.65 | 0.000,299 | 0.001,196 | 0.001,212 | 0.003,394 | 0.000,898 | $0.003,591$ |
| 100 | 144 | 1.39 | 0.000,288 | 0.001,152 | 0.001,367 | 0.003,741 | 0.000,910 | $0.003,640$ |

Note: The total number of trees in the experiment is assumed to be 200 .
Case 1 Environmental variation negligible.
Case 2 Environmental variation independent of plot size.
Case 3 Genetic variation negligible.

TABLE 8. VARIANCES OF TREATMENT MEANS BASED ON DIFFERENT NUMBERS OF RECORDED TREES IN A PLOT
(Minimum values in bold type. The 'guard' zone is given by one boundary row on either side of the recorded rows and two trees at either end of the recorded rows).

| No. of recorded trees $=n$ | Number of trees in a $\mathrm{plot}=N$ | No. of replications$r=\frac{200}{N}$ | $\frac{\text { Variance }}{\bar{x}^{2}}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Girth | Case 1 Girth increment | Girth | Case 2 Girth increment | Girth | Case 3 Girth increment |
| 1 | 15 | 13.33 | 0.003,001 | 0.012,004 | 0.003,113 | 0.012,273 | 0.003,001 | 0.012,003 |
| 4 | 24 | 8.33 | 0.001,200 | 0.004,800 | 0.001,381 | 0.005,234 | 0.001,698 | 0.006,791 |
| 9 | 35 | 5.71 | 0.000,778 | 0.003,112 | 0.001,041 | 0.003,744 | 0.001,348 | 0.005,391 |
| 16 | 48 | 4.17 | 0.000,600 | 0.002,400 | 0.000,959 | 0.003,261 | 0.001,199 | 0.004,796 |
| 25 | 63 | 3.17 | 0.000,505 | 0.002,020 | 0.000,978 | 0.003,155 | 0.001,129 | 0.004,515 |
| 36 | 80 | 2.50 | 0.000,444 | $0.001,776$ | 0.001,044 | 0.003,218 | 0.001,089 | 0.004,355 |
| 49 | 99 | 2.02 | 0.000,404 | 0.001,616 | 0.001,147 | $0.003,399$ | 0.001,069 | $0.004,277$ |
| 64 | 120 | 1.67 | 0.000,374 | 0.001,496 | 0.001,272 | 0.003,653 | 0.001,058 | 0.004,234 |
| 81 | 143 | 1.40 | $0.000,353$ | $0.001,412$ | 0.001,424 | 0.003,982 | 0.001,058 | 0.004,232 |
| 100 | 168 | 1.19 | 0.000,336 | 0.001,344 | 0.001,597 | 0.004,370 | 0.001,063 | 0.004,251 |

Note: The total number of trees in the experiment is assumed to be 200.
Case 1 Environmental variation negligible.
Case 2 Environmental variation independent of plot size.
Case 3 Genetic variation negligible.

## R. Narayanan: Optimum Number of Trees Per Plot

The assumptions made in the three cases discussed here regarding the genetic and environmental components of the total variation, may not be true in general. As pointed out by Pearce (1955), the relative importance of these two sources of variation will depend very much upon the circumstances and in general neither can be ignored. In the absence of any data from uniformity trials, it is not possible to ascertain the relative importance of these two sources of variation.

One should also take into account the many practical problems involved in the fixing of an adequate plot size. The use of smaller plots must be discounted because of the uneven distribution of tree losses, while very big plots create difficulties with regard to the overall size of the experiment, with consequent problems concerning the site blocking of treatments, etc. Again, the amounts of land and material needed to 'guard' the experimental treatments are relatively greater for a small plot than for a large one.

Paardekooper (1964) has studied tree variation within plots and plot variation in a number of clone trials and it is seen that clonal tree variation ( 2 to 3 years after budding) is smaller than seedling tree variation. The plot variation in the different trials is of an order similar to that in the experiment discussed here.

Thus there is no optimum plot size which can be applied in all situations. Practical considerations, coupled with the above theoretical ones, indicate that about 35 to 45 recorded trees in a plot should be sufficient
for manuring experiments on immature Hevea brasiliensis. After normal losses due to thinning and natural causes in the course of the experiment, the number will be about 25 to 35 trees per plot.

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