

Predicting the Performance of Bonded Filled Rubber Mounts in Compression Using Data Measured in a Simple Shear Deformation[†]

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The problem of predicting the stiffness of bonded, carbon black filled rubber mounts is difficult because of the problems associated with geometric non-linearity and material characterisation. It is proposed that alternative simple forms of stored energy function can be employed to model the non-linear material properties. For carbon black filled materials it is apparent that it is possible to obtain the relevant materials data from just a single materials test made in any of a number of simple homogeneous deformation modes. The approach is validated by taking materials data for simple shear measurements and using the data to predict the behaviour of compression mounts with a range of shape factors.

The advent of new generation high speed computers and the availability of finite element analysis (FEA) computer application packages such as ABAQUS, MARC and ANSYS have paved the way towards a speedier method of analysing complex design problems. However, it is apparent that several issues such as materials characterisation still need to be addressed in the analysis of rubber components using finite element method. Several over-views on the use of FEA in elastomer components are available^{1,2}.

Materials Characterisation

Characterising the stress-strain behaviour of filled rubbers for use in finite element analysis has been dealt with by many workers such

as Charlton *et al.*³ Numerous constitutive theories have been used to model the non-linear behaviour of the rubber. The parameters characteristic of the materials are normally determined from the tensile or compression, pure shear, simple shear or bi-axial experiments. Most workers have adopted a strain energy approach. In this case the strain energy function is either expressed as a polynomial function of the strain invariants, $W = W(I_1, I_2, I_3)$, or directly in terms of principal stretch ratios, $W = W(\lambda_1, \lambda_2, \lambda_3)$. The strain invariants I_1, I_2 and I_3 can be defined as:

$$\begin{aligned} I_1 &= \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \\ I_2 &= \lambda_1^2 \lambda_2^2 + \lambda_2^2 \lambda_3^2 + \lambda_3^2 \lambda_1^2 \\ I_3 &= \lambda_1^2 \lambda_2^2 \lambda_3^2 \end{aligned} \quad \dots 1$$

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where $\lambda_1, \lambda_2, \lambda_3$ are the extension ratios in the three perpendicular directions of the principal axes.

Both of these two forms of strain energy function for solid rubber are options available in ABAQUS and are called the polynomial and Ogden forms, respectively. The most general polynomial form is based on the theory of rubber elasticity developed by Rivlin⁴ and is expressed in terms of the strain invariants as:

$$W = \sum_{i=1}^N C_{ij} (\bar{I}_1 - 3)^i (\bar{I}_2 - 3)^j + \sum_{i=1}^N \frac{1}{D_i} (J_{el} - 1)^2 \quad \dots 2$$

While the Ogden form is expressed in terms of the principal stretch ratios as:

$$W = \sum_{i=1}^N \frac{2\mu_i}{\alpha_i^2} (\bar{\lambda}_1^{\alpha_i} + \bar{\lambda}_2^{\alpha_i} + \bar{\lambda}_3^{\alpha_i} - 3) \gamma + \sum_{i=1}^N \frac{1}{D_i} (J_{el} - 1)^2 \quad \dots 3$$

where $\bar{\lambda}_i = \frac{\lambda_i}{\sqrt[3]{J}}$ are the deviatoric principle

stretches. In both of the above expressions N is the order of the energy function. The coefficients C_{ij} and μ_i determine the deviatoric (shear) behaviour, while the D_i introduce compressibility. J_{el} is the elastic volume ratio.

Apart from the two general forms above, users are also allowed to define other hyperelastic material models in ABAQUS by the incorporation of a FORTRAN sub-routine UHYPER in the input file. UHYPER requires the coding of the strain energy function and the first and second derivatives with respect to I_1, I_2 and I_3 .

It is noted that both the Rivlin and Ogden material models are power series expansions, so it is apparent that it is possible to fit experimental stress strain data to any desired degree of precision provided that sufficient terms are taken. It is a common fallacy to assume that

more terms makes the model more accurate. In reality the introduction of more terms allows the regression analysis to fit the experimental errors better. This frequently leads to unstable functions which predict unrealistic behaviour outside the range of the experimental data as shown by James and Green⁵.

Treloar⁶ noted that in fact the Rivlin and Ogden formulations are essentially equivalent, the choice of one over the other is just a matter of convenience. One difficulty of either method is that they contain arbitrary fitting constants of unknown physical significance. Therefore the problem of materials characterisation reduces to one of a simple curve fitting exercise and it is difficult to know when a model of the required accuracy has been achieved. Recent work has shed new light on this process and on the underlying physics of these models. This has allowed the recent development of realistic and simplified materials models.

Kawabata and Kawai⁷ have pointed out that in order to fit test data, using say the Rivlin power series requires the use of multiple sets of test data obtained from multiple deformation modes. Gregory⁸ however noted that the coefficients for the (I_1-3) terms were much greater than those for the (I_2-3) terms. This led to Yeoh⁹ proposing for filled elastomers the proposition that $W = f(I_1 - 3)$ only. This has been verified for a range of carbon black filled materials by Davies *et al.*¹⁰

This simplification dramatically improves the materials characterisation problem as materials tests are now only required in a single deformation mode. If the case of simple shear is taken, Treloar⁶ showed that expressed in terms of I_1 , only the shear stress, τ , is related to shear strain, γ , as shown below:

$$\frac{\partial W}{\partial I_1} = \frac{1}{2} \frac{\tau}{\gamma} \quad \dots 4$$

also that the shear strain, γ , is related to I_1 by:

$$\gamma^2 = (I_1 - 3) \quad \dots 5$$

The measured functional dependence of $\frac{\partial W}{\partial I_1}$ on $(I_1 - 3)$ can be seen in *Figure 1*, for four different carbon black filled natural rubber compounds selected from the MRPRA Data Sheets¹¹, by plotting $\frac{\partial W}{\partial I_1} (= \frac{G_{ch}}{2})$ against $\gamma^2 = (I_1 - 3)$. It is obvious from the shape of this relationship that an accurate fit of the experimental behaviour is *not* most readily achieved using the power series approach of Rivlin or Ogden.

Plotting the graphs on a log $(\frac{G_{ch}}{2})$ against log $(I_1 - 3)$ axes, reveals that the data can over the engineering strain range be approximated by a power law relationship such as:

$$G_{ch} = A\gamma^n \text{ or } \frac{\partial W}{\partial I_1} = \frac{A}{2}(I_1 - 3)^{-n/2} \quad \dots 6$$

Davies *et al.*¹⁰ integrated this function and proposed a stored energy function for use over most of the engineering strain range, equivalent to a maximum shear strain of about 100%:

$$W = \frac{A}{2(1-n/2)} (I_1 - 3)^{(-n/2)} \quad \dots 7$$

Physically A is a measure of the 100% shear strain modulus and n is a measure of the small strain non-linearity.

If it is decided to predict the behaviour in the higher strain upsweep regions of the curve caused by the finite extensibility of the rubber network, then an additional power term in $(I_1 - 3)$ can be added. The new proposed strain energy function is therefore:

$$W = \frac{A}{2(1-n/2)} (I_1 - 3)^{(-n/2)} + K (I_1 - 3)^2 \quad \dots 8$$

The lines shown in *Figure 2* represent the best fit relationships fitted for A , n and K with the fitting parameters selected for the MRPRA Data¹¹ shown in *Table 1*.

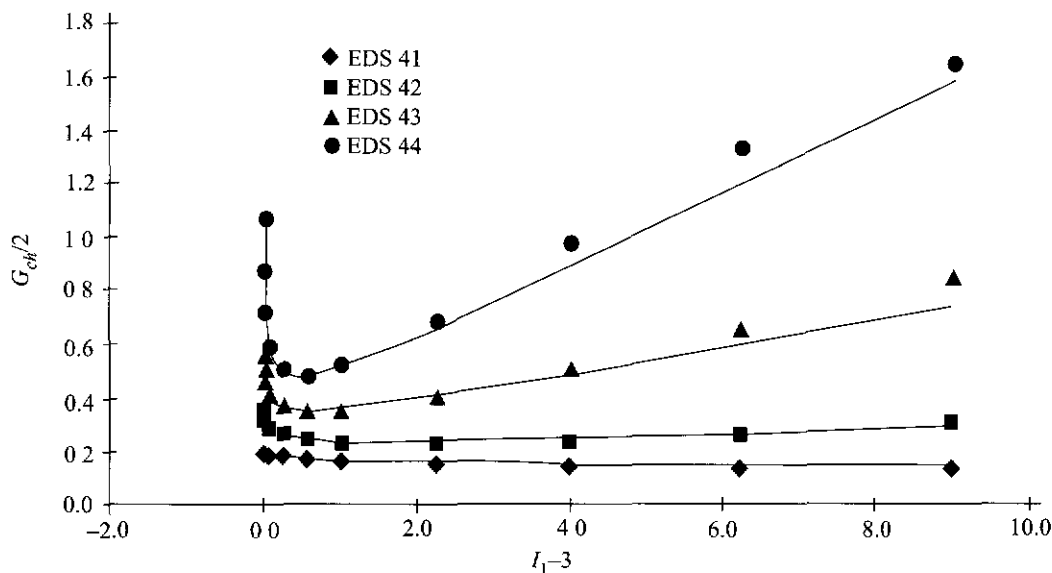


Figure 1 Plot of $G_{ch}/2$ versus $(I_1 - 3)$.

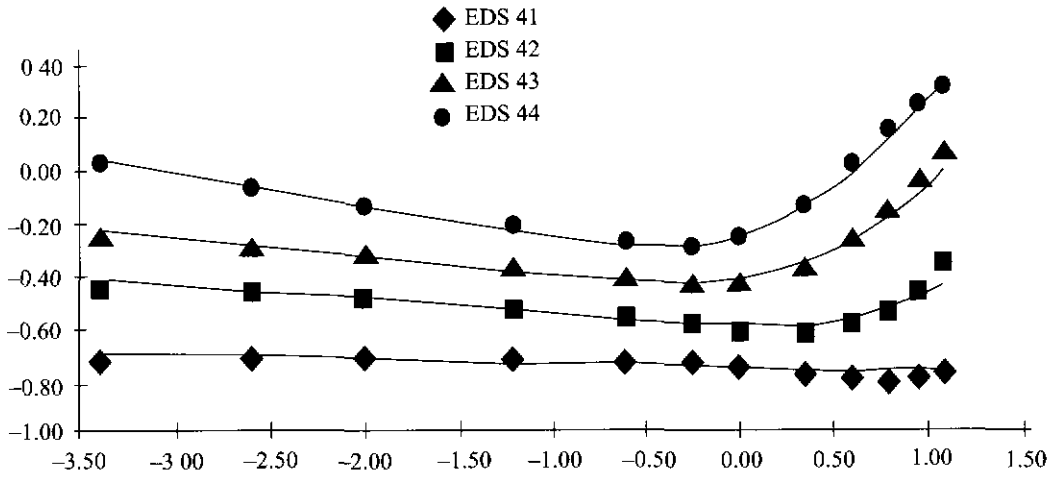


Figure 2 Curve fitting of experimental shear data with parameters A, n, C and K.

One final modification is required before the strain energy function can be implemented in a Finite Element Code. It is observed that as it stands, at very small strains the modulus becomes very high. In fact the models lock at zero strain as the modulus is infinite. To compensate for this a small strain correction is introduced. The approach adopted is supported by measurements taken from Payne and Whittaker¹² for small strains down to 0.001% strain. They showed a finite small strain value for the modulus, G_o .

A new term, C, which is the strain at which the modulus tends to approach to this finite small strain value can therefore be introduced:

$$G_o = AC^n \quad \dots 9$$

This can be implemented into the stored energy function in Equation 8 as:

$$W = \frac{A}{2(1-n/2)} (I_1 - 3 + C^2)^{(1-n/2)} + K(I_1 - 3)^2 \quad \dots 10$$

This function can be implemented in ABAQUS using the UHYPER sub-routine shown in Appendix 1. This requires the definition of W as well as its first and second derivatives:

$$UI1(1) = \frac{\partial W}{\partial I_1} = \frac{A}{2} (I_1 - 3 + C^2)^{-n/2} + 2K(I_1 - 3) \quad \dots 11$$

$$UI2(1) = \frac{\partial^2 W}{\partial I_1^2} = -\frac{An}{4} (I_1 - 3 + C^2)^{-1-n/2} + 2K \quad \dots 12$$

TABLE 1. CURVE FITTING PARAMETERS FOR THE FOUR BLACK FILLED COMPOUNDS

Parameters	EDS 41	EDS 42	EDS 43	EDS 44
n	0.056	0.136	0.1698	0.28
A	0.332	0.458	0.611	0.743
K	0.0002	0.006	0.028	0.074
C	0.003	0.003	0.003	0.003

Analysis of Bonded Compression Mounts

To test the validity of the proposed strain energy function the data is used to predict the behaviour of a different non-homogeneous deformation mode. The geometry chosen is that of a cylindrical bonded rubber mount, shown in *Figure 3*.

A variety of different shape factors are analysed where the shape factors, S , is defined as the loaded area divided by the force free area. For a cylindrical geometry this reduces to:

$$S = \frac{R}{4H} \quad \dots 13$$

The rubber is modelled using axisymmetric elements of type CAX4H. The meshed area is represented by the hashed section. Symmetry considerations allow the reduction of the problem to that of a half height block. A typical half height model is shown in *Figure 4*. The dimensions of the 4-node hybrid element corresponding to the shape factors of the disc are shown in *Table 2*.

Each of the models is analysed subjected to incremental compressive deflections until the model either collapses or a maximum equivalent to a 20% reduction in height H is achieved. These results are compared with measured test data also available at 5%, 10%, 15% and 20% values in the MRPRA Data Sheets¹¹. The data is expressed as an apparent compression chord modulus, E_{ch} , which is calculated from:

$$E_{ch} = \frac{F/A_0}{x/H} \quad \dots 14$$

where F is the total reaction force, A_0 is the loaded area, x is the deflection and H is the initial half height of the mount.

RESULTS

The measured and the finite element results are compared in *Figure 5*. The solid lines show the finite element predictions of the apparent chord modulus, E_{ch} , obtained by analysing the six different geometries for each of the four different materials over a range of deforma-

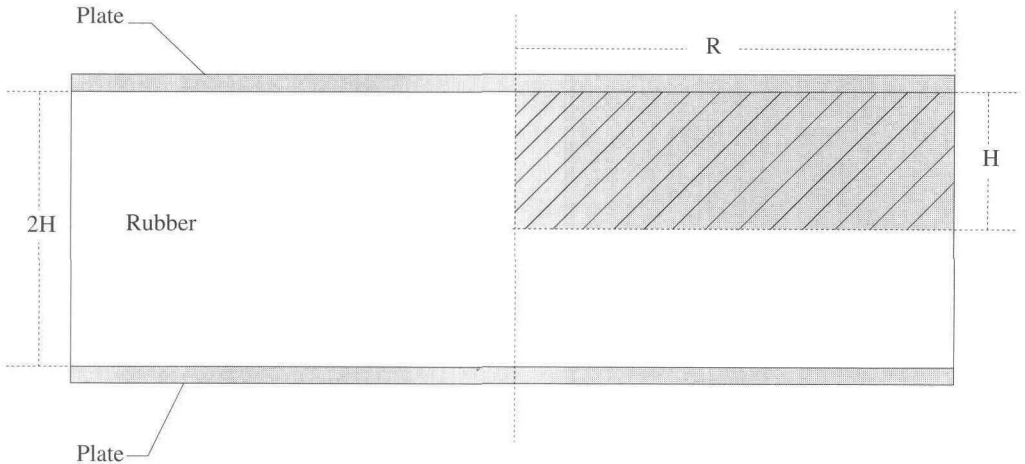


Figure 3. Schematic representation of the cylindrical rubber mount. The shaded area represents the axisymmetric model used to perform the analysis.

TABLE 2. DIMENSIONS OF THE HALF HEIGHT AXISYMMETRIC FINITE ELEMENT MODELS FOR VARIOUS SHAPE FACTORS

Shape factor	0.5	1	2	3	5	8
R / mm	48	48	48	48	48	48
H / mm	24	12	6	4	2.4	1.5

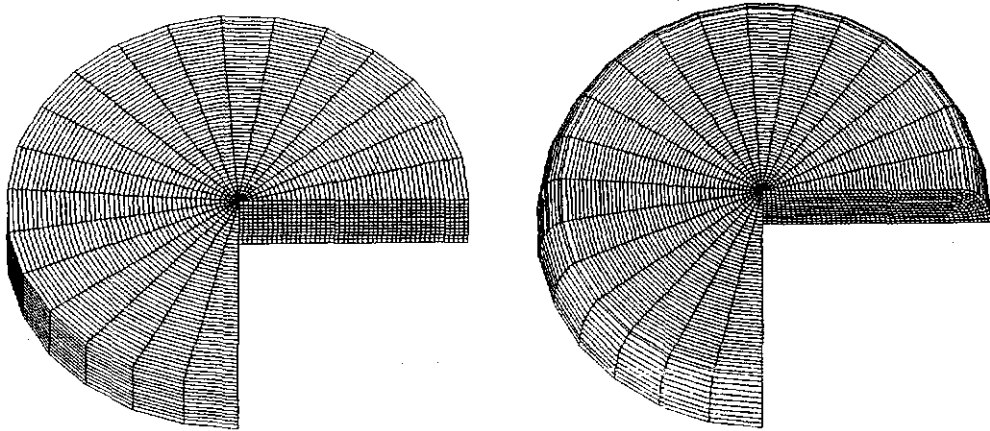


Figure 4. A typical finite element model used in the analysis, with the deformed shape shown on the right. The model shown has a shape factor of 1.

tions. The points show the actual measured data shown in the MRPRA data sheets¹¹ for a range of different deformations, materials and geometries. The first observation is that for all the low shape factors the agreement is in general excellent. As the shape factor increases then the agreement in the upsweep region is less reliable.

It is interesting to ascertain as to whether it is the material or geometric non-linearity that causes the upsweep in the apparent chord modulus as the level of deformation increases for the moderate and large shape factor mounts. Figure 6 shows a plot of EDS 44 with the material upsweep behaviour

K included and with the parameter set to zero. In this graph it is clear that the apparent upsweep is much higher for the material with a realistic K parameter governing the increase in modulus due to the effect of the finite extensibility of the rubber network. Therefore the increase in the analytical results at higher deformations is not due to geometric effects but principally to the material non-linearity.

It is worth observing that at the higher shape factors the predictions are not very accurate at the higher deformations, as the finite element analysis seems to over-estimate the stiffness of the components.

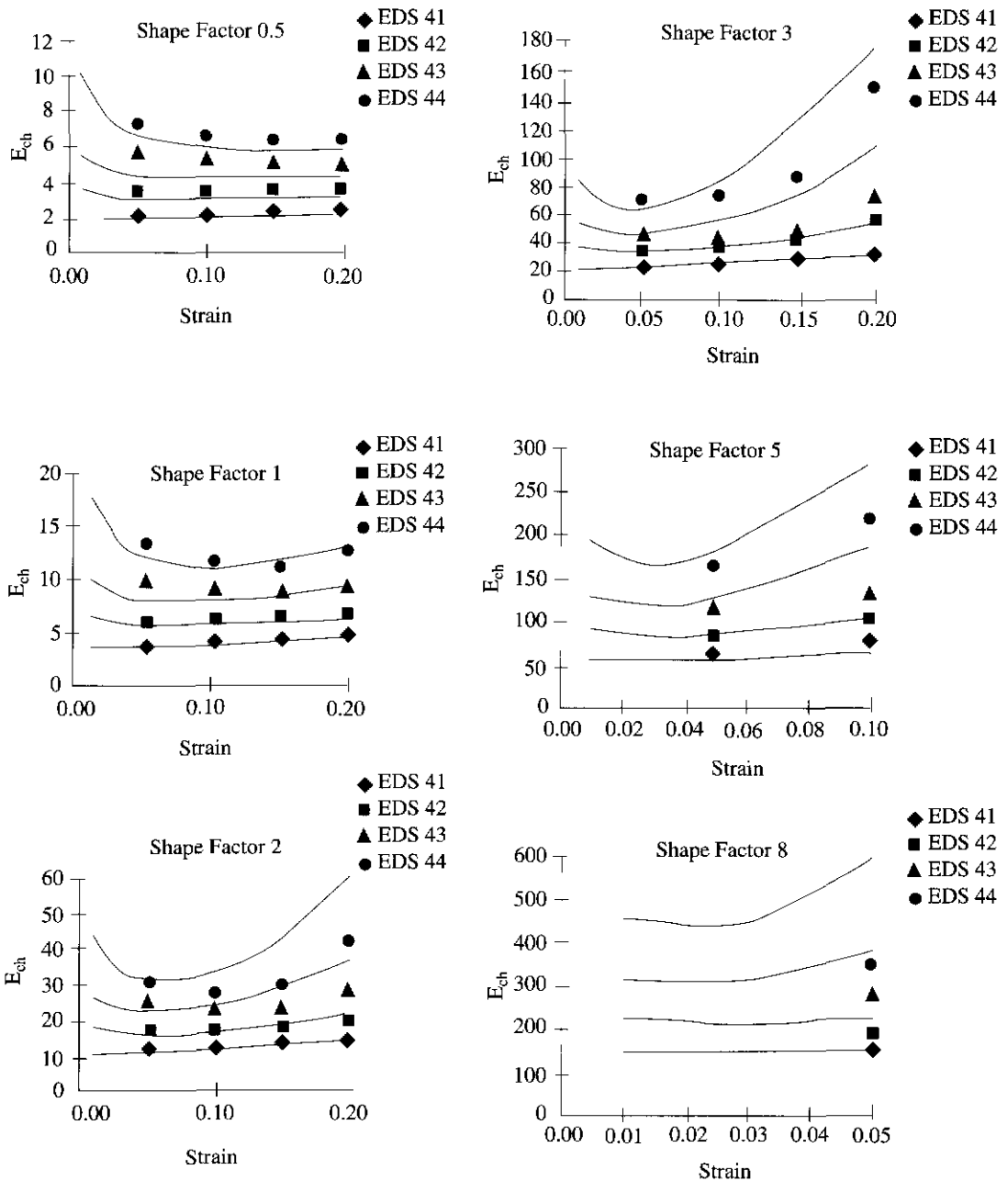


Figure 5. Graphs of E_{ch} against strain plotted for both the finite element analysis results shown as the solid lines and the measured results shown as the data points. These results are shown for six different values of shape factor ranging from 0.5 to 8 for four different filled materials.

Shape Factor 8

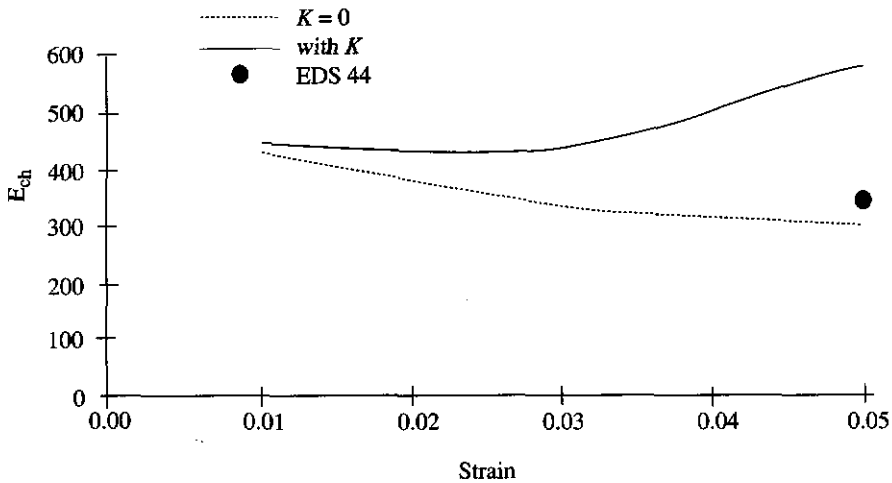


Figure 6. Plot showing the effect of K on the E_{ch} values calculated by FEA.

This may be due to bulk compressibility. These effects are examined in more detail by re-examining the results for the highest modulus material EDS 44 with a shape factor of eight at the highest deformation. The compression block is modelled by introducing a bulk compressibility of 2000 MPa taken from the MRPRA data sheets¹¹, and it is observed that the calculated finite element value of E_{ch} drops by 29%. This is shown by the arrow and the point on the final graph in Figure 7. Thus it is anticipated that the large discrepancies can be largely accounted for by the incorporation of the bulk modulus effects.

The decision to use the strain energy function with or without K depends on the strain regions where the application will be used as well as whether an upsweep in the strain energy function occurs. Muhr *et al.*¹³ derived a relationship between an average shear strain, $\bar{\gamma}$, compressive strain, e_c , and shape factor, S , for a bonded rubber disc in compression and is expressed as follows:

$$\bar{\gamma} \approx e_c (3 + 6S^2)^{1/2} \quad \dots 15$$

It is apparent therefore for a mount having a large shape factor subjected to a small compressive strain will be experiencing an equivalent high average shear strain. Thus a rubber mount having a shape factor of eight subjected to a compressive strain of 5% will be experiencing an average shear strain of about 100%. Referring to Figure 2, for this value of average shear strain, the strain energy function curves representing the EDS 44 and EDS 43 compounds are already showing the upsweep phenomena. It is observed that the user must take precautions when predicting the performance of large shape factor compression mounts at apparently small compressive strains as these may in fact be very large equivalent average shear strains.

It is also worth noting that the accurate determination of the high stiffness of bonded blocks of large shape factor, particularly made from filled rubbers, is notoriously difficult

Shape Factor 8

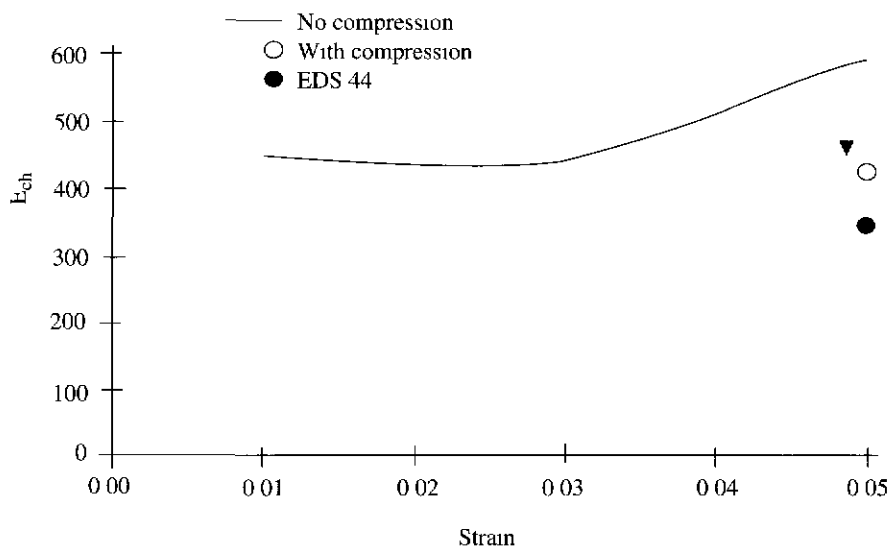


Figure 7 Plot showing the decrease in the E_{ch} value calculated by FEA when compressibility is taken into account

because the compliance of the measuring equipment may not be negligible compared with that of the specimen itself. Thus the agreement we find between experiment and theory for blocks of these shapes can be considered quite satisfactory.

CONCLUSION

It is apparent that the proposed simple stored energy function with data fitted in one homogeneous deformation mode, can be applied to alternative non-homogeneous deformation modes and give an accurate representation of the deformation behaviour. This is validated by the good correlation between measured stiffness data and finite element predicted stiffness data for the compression mounts. The only problem encountered is the difficulty in

predicting the bulk compressibility effects if the stored energy function was recast in terms of the reduced strain invariants and a bulk compression term added, then it would appear that the function could be used to predict the behaviour of filled materials under any deformation mode. This dramatically simplifies the materials characterisation involved in predicting the behaviour of filled elastomeric materials using finite element analysis.

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APPENDIX 1.

*USER SUB-ROUTINES

```
SUB-ROUTINE UHYPER (BI1, BI2, AJ, U, UI1, UI2, UI3, TEMP, INOEL,
CNAME)
C
C INCLUDE 'ABA_PARAM.INC'
C
CHARACTER*8 CMNAME
DIMENSION UI1(3), UI2(6), UI3(6)
UI1(2) = 0
UI1(3) = 0
UI2(2) = 0
UI2(3) = 0
UI2(4) = 0
UI2(5) = 0
UI2(6) = 0
UI3(2) = 0
UI3(3) = 0
UI3(4) = 0
UI3(5) = 0
UI3(6) = 0
AJ = 1.0
SMOD = 0.743
ALPHA = 0.28
SMSTC = 0.003
FULL = 0.074
ETA = (1.0 - (ALPHA/2.0))
DELTA = ((SMSTC**2.0) - 3.0)
OMEGA = SMOD/(2.0* (ETA))
U = OMEGA* ((BI1 - DELTA)** ETA) + FULL* (BI1 - 3.0)** 2.0
UI1(1) = OMEGA* ETA* ((BI1 + DELTA)** (ETA - 1.0)) - 2.0* FULL* (BI1 - 3.0)
UI2(1) = OMEGA* ETA* (ETA - 1.0)* ((BI1 + DELTA)** (ETA - 2.0)) + 2.0* FULL
RETURN
END
```