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Damping Behaviour of Strained Rubber Strips in Free Transverse and Longitudinal Oscillations

T.J. POND* AND A.G. THOMAS**

Considerable differences have been observed between the decay of oscillations occurring in a stretched rubber strip when vibrating in a longitudinal and transverse mode. A theory considering changes in strain and elastic stored energy is given which describes this effect. The design and construction of the apparatus to investigate quantitatively the difference in damping between the two oscillation modes are described. Results are presented using three rubber compounds covering the range of damping values usually found in rubbery materials. Measurements have shown that differences in damping between the two modes of oscillation can vary from approximately ten to one upto the order of a thousand to one.

It has been observed that the transverse vibrations of a stretched rubber strip can continue for a relatively large number of cycles compared with that of longitudinal oscillations of, for example, a weight suspended by a similar strip. The rubber in the transverse oscillation case, stores all the kinetic energy in the form of elastic energy at the point of maximum amplitude and gives it up as kinetic energy at the mid-point. Each element of the rubber undergoes a strain-energy cycle which is similar to that experienced by the longitudinal oscillations. It may not, therefore, be obvious why the decay of the oscillations in the two modes can be quite different. The important difference between the two modes is that in the transverse case, when deflecting the strip, all the elements in the strip increase in strain so that a relatively small strain cycle gives a large elastic stored energy change. In the longitudinal case, some elements increase in strain while at the same time others decrease, so that a larger strain amplitude in the various elements is required to achieve a similar net change in total elastic energy. In the case of a strip supporting a weight against gravity the compensating energy changes are due to the gravitational potential of the weight, but the general principles remain the same.

A theory considering changes in strain and elastic stored energy for the two cases of transverse and longitudinal oscillations is given here. An apparatus has been designed and constructed to investigate these effects quantitatively by measuring the damping of three rubber compounds, subjected to a range of fixed strains, in the two oscillation modes.

THEORY

Transverse Oscillations

Consider a uniform strip of rubber initially of length $2l_o$ and extended between rigidly fixed grips to a length $2l$, so that $\lambda = l/_{l_o}$ as shown in *Figure 1*. The initial cross sectional area is A. The mid-point of the strip is displaced sideways by x. The strip is considered sufficiently thin; the elastic energy associated with bending at the clamps and at the centre point can be neglected.

The increase in length of the strip is given by $(\sqrt{l^2 + x^2} - l)$ so that the increase in strain relative to the initially deformed state $(2l)$ is given by

$$\Delta e_1 = \frac{\sqrt{l^2 + x^2} - l}{l} = \sqrt{1 + \frac{x^2}{l^2}} - 1 \quad \ldots 1$$

*Malaysian Rubber Producers' Research Association (MRPRA), Hertford, SG 13 8NL, United Kingdom

**Consultant, MRPRA

x = Sideways deflection

$2l$ = Initial strained length

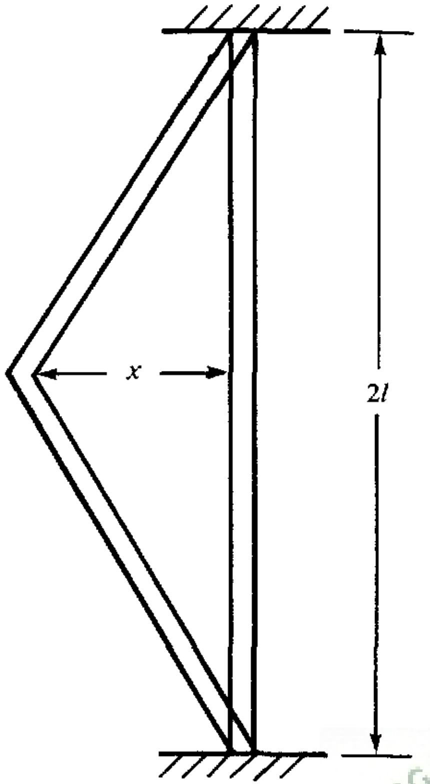

Figure 1. Test-piece — transverse configuration.

This definition of Δe_1 is used as the energy loss when taken around a strain cycle and this loss is more likely to be determined by this quantity than by the increase in strain relative to the initial, unstressed length of the strip However, no precise dependence of the loss modulus on strain needs to be assumed in the subsequent analysis so this definition is in no way restrictive. If the strip is deflected sideways by a distance x and returned to its original position, the strain in each element will be cycled through $\pm \frac{1}{2}\Delta e_1$ as given by *Equation 1.*

The energy loss per unit volume in one such cycle will be given by

$$\Delta L_1 = \pi E'' \left(\frac{\Delta e_1}{2}\right)^2$$

$$= \frac{\pi}{4} E'' (\Delta e_1)^2 \qquad \ldots 2$$

where E'' is the extension loss modulus. During a complete oscillation of the strip, x becomes negative and a similar strain cycle will be undergone, leading to an equal energy loss. Thus for a complete oscillation of the strip by $\pm x$, the energy loss will be given by

$$2\Delta L_1 = \frac{\pi}{2} E'' (\Delta e_1)^2 \qquad \ldots 3$$

For $x << l$, *Equation 1* gives

$$\Delta e_1 = \frac{1}{2} \frac{x^2}{\lambda^2 l_o^2} \qquad \ldots 4$$

The increase in the elastic stored energy per unit volume on sideways deflection by x is given to a first order by

$$\Delta u_1 = F . \Delta\lambda \qquad \ldots 5$$

where F is the stress in the rubber, referred to the initial unstrained cross-section, and $\Delta\lambda$ is the increase in extension ratio due to the deflection. *Equation 5* is valid as long as $(\lambda - 1) >> \Delta\lambda$, that is λ is significantly greater than 1. Then

$$\Delta\lambda = \sqrt{\lambda^2 + \frac{x^2}{l_o^2}} - \lambda \qquad \ldots 6$$

giving, for small x/l_o,

$$\Delta\lambda = \frac{\lambda}{2} \frac{x^2}{\lambda^2 l_o^2} = \frac{x^2}{2\lambda l_o^2} \qquad \ldots 7$$

and from *Equation 5*

$$\Delta u_1 = \frac{F}{2\lambda} \left(\frac{x}{l_o}\right)^2 \qquad \ldots 8$$

For the energy loss per cycle, $2\Delta L_1$, *Equations 3* and *4* give

$$2\Delta L_1 = \frac{\pi}{2} E'' \left[\frac{1}{4} \cdot \frac{x^4}{\lambda^4 l_o^4}\right] = \frac{\pi}{8} E'' \frac{x^4}{\lambda^4 l_o^4}. \qquad \ldots 9$$

Thus the fractional loss in energy per cycle is

$$\frac{2\Delta L_1}{\Delta u_1} = \frac{\pi}{8}\frac{E''x^4}{\lambda^4 l_o^4} \cdot \frac{2\lambda l_o^2}{Fx^2}$$

$$= \frac{\pi}{4}\frac{E''}{F\lambda^3}\frac{x^2}{l_o^2} \qquad \ldots 10$$

This shows that the fractional energy loss per cycle depends on the amplitude of oscillation x so that the decay of a free oscillation is not expected to follow the usual logarithmic form with number of cycles.

In the case of free oscillation, the decay in amplitude for successive oscillations can be derived readily with the assumption, normally valid, that the fractional loss of energy per cycle is small. The reduction in amplitude Δx per oscillation will be given by

$$\Delta x = 2\Delta L_1 / \frac{d(\Delta u_1)}{dx} \qquad \ldots 11$$

From *Equation 8*

$$\frac{d(\Delta u_1)}{dx} = \frac{Fx}{\lambda l_o^2} \qquad \ldots 12$$

and thus from *Equations 9* and *11*

$$\Delta x = \frac{\pi}{8} E'' \frac{x^4}{\lambda^4 l_o^4} \frac{\lambda l_o^2}{Fx}$$

$$= \frac{\pi E'' x^3}{8F\lambda^3 l_o^2} \qquad \ldots 13$$

Hence, the fractional decay in amplitude of each cycle is given by

$$\frac{\Delta x}{x} = \frac{\pi}{8F} \cdot \frac{E''}{\lambda^3} \cdot \left(\frac{x}{l_o}\right)^2 \qquad \ldots 14$$

This shows an unusual decay behaviour in that the damping commonly found for simple harmonic motion is such that the fractional decay is independent of amplitude whereas in this case it varies with its square.

If the decrease in amplitude per cycle is small, *Equation 13* can be written in differential form for the decay in amplitude dx in dn cycles, thus

$$dx = \frac{\pi}{8}\frac{E''}{F\lambda^3 l_o^2} x^3 \,.\, dn$$

giving

$$\frac{dx}{x^3} = \frac{\pi}{8}\frac{E''}{F\lambda^3 l_o^2} dn \,. \qquad \ldots 15$$

Integrating from an initial amplitude x_o at $n = o$, the amplitude x after n cycles is given by

$$\frac{1}{x^2} = \frac{\pi}{4}\frac{E''}{F\lambda^3 l_o^3} n + \frac{1}{x_o^2} \qquad \ldots 16$$

Thus a plot of $\frac{1}{x^2}$ *versus* n should give a straight line of slope $\frac{\pi}{4}\frac{E''}{f\lambda^3 l_o^2}$.

An indication of the rate of amplitude decay to be expected can be obtained by taking the case of a rubber obeying the statistical theory stress-strain relation, *viz:*

$$F = \frac{E_o'}{3}\left(\lambda - \frac{1}{\lambda^2}\right) \qquad \ldots 17$$

where E_o' is the Young's modulus for small strains.

From *Equation 14*

$$\frac{\Delta x}{x} = \frac{3\pi E''}{8 E_o'}\left(\frac{x}{l_o}\right)^2 \frac{1}{(\lambda^4 - \lambda)}. \qquad \ldots 18$$

A typical, not very resilient rubber, might have a value of E''/E_o' [= tan (loss angle)] of 0.1. Thus for $\lambda = 2$ and $x/l_o = 0.1$, *Equation 18* gives $\Delta x/x$ as 8.4×10^{-5}. This very small quantity illustrates the low apparent damping that would be expected for transverse oscillations, in qualitative agreement with observations.

The very small damping value illustrated does indicate however that there may be experimental difficulties in making quantitative measurements of the damping as other sources of energy loss, such as air damping and losses associated with the bending of the strip near the clamps, are likely to complicate the results.

Longitudinal Oscillations

The corresponding longitudinal oscillation is illustrated in *Figure 2*. The rubber strip, of strained length $2l$, and initial length $2l_o$, is considered to be deformed by movement of its mid-point, vertically, by an amount $\pm x$. For small values of x, the change in the total elastic energy of the system is given by $\frac{1}{2}fx$ where f is the net force on the centre point to move it a distance x. The force in the upper half of the strip will decrease by the same amount as the increase in the lower half so that

$$f = \frac{2x}{l_o}\frac{\partial F}{\partial \lambda} \cdot A_o \qquad \ldots 19$$

where A_0 is the unstrained cross-sectional area of the strip and $\partial F/\partial\lambda$ is its incremental stiffness.

x = Deflection

$2l$ = Initial strained length

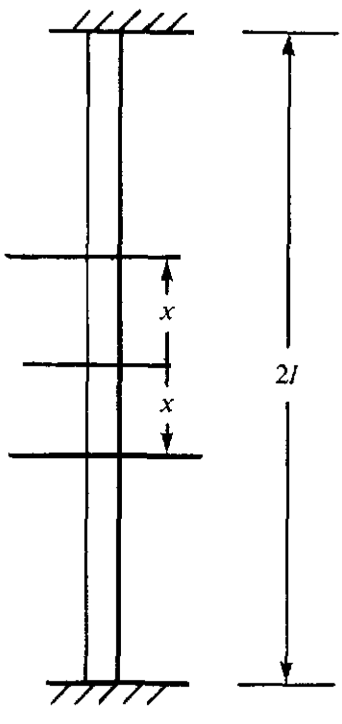

Figure 2. Test-piece — longitudinal configuration.

Thus, the increase in the stored elastic energy is given by

$$\Delta U_2 = \frac{1}{2} x \cdot \frac{2x\partial F}{l_o \partial\lambda} A_o$$

$$= x^2 \frac{A_o}{l_o}\frac{\partial F}{\partial\lambda} \qquad \ldots 20$$

The change in strain relative to the deformed state due to the movement x is given by

$$\Delta e_2 = \frac{x}{l}$$

$$= \frac{x}{\lambda l_o} \qquad \ldots 21$$

so that the loss of energy on taking the mid-point through a complete cycle is given by

$$\Delta L_2 = 2\pi E'' \left(\frac{x}{\lambda l_o}\right)^2 \cdot l_o A_o \qquad \ldots 22$$

The loss modulus E'' has the same significance as that used in the analysis for the transverse oscillations *(Equation 2)*. The fractional energy loss per cycle is given from *Equations 20* and *22* by

$$\frac{\Delta L_2}{\Delta U_2} = 2\pi E'' \left(\frac{x}{\lambda l_o}\right)^2 \frac{l_o a_o l_o}{x^2 A_o(\partial F/\partial\lambda)}$$

$$= \frac{2\pi E''}{\lambda^2(\partial F/\partial\lambda)} \qquad \ldots 23$$

Equation 23 shows that the fractional loss is independent of the amplitude x, as of course is usually the case with damped sinusoidal oscillations and in contrast with the result for the transverse oscillations.

The reduction in amplitude per cycle is given by

$$\Delta x = \Delta L_2 / \frac{\partial(\Delta U_2)}{\partial x} \qquad \ldots 24$$

From *Equation 20*

$$\partial (\Delta U_2)/\partial x = \frac{2xA_o}{l_o} \partial F/\partial\lambda \qquad \ldots 25$$

and from *Equation 22*

$$\Delta x = 2\pi E'' \frac{x^2}{\lambda^2 l_o^2} \frac{l_o A_o l_o}{2xA_o \partial F/\partial\lambda}$$

$$= \frac{\pi E'' x}{\lambda^2 \partial F/\partial\lambda} \qquad \ldots 26$$

giving

$$\frac{\Delta x}{x} = \frac{\pi E''}{\lambda^2(\partial F/\partial\lambda)} \qquad \ldots 27$$

For the case in which the stress-strain relation can be represented as strain proportional to true stress (an approximation which is roughly obeyed up to moderate strains), *viz.*

$$F = E'\ (1 - 1/\lambda), \qquad \ldots 28$$

$\partial F/\partial\lambda = E'/\lambda^2$ and *Equation 27* becomes

$$\frac{\Delta x}{x} = \frac{\pi E''}{E'} \qquad \ldots 29$$

which gives independence of strain. Of course, E'' and E' may well depend on frequency, and the natural frequency of oscillation of the strip with a central mass will generally depend on the strain, so allowance may have to be made for this in any experimental test.

Summarising the results, for transverse damping, the fractional reduction in the amplitude of oscillation per cycle $(\Delta x/x)T$, is given by

$$(\Delta x/x)_T = \frac{\pi}{8} \frac{E''}{F\lambda^3} \left(\frac{x}{l_o}\right)^2 \qquad \ldots 30$$

For the longitudinal damping case, the fractional decrement $(\Delta x/x)_L$ is given by

$$(\Delta x/x)_L = \frac{\pi}{\lambda_o^2} \frac{E''}{(\partial F/\partial\lambda)} \qquad \ldots 31$$

Thus, the ratio of the damping in the two oscillation modes is given by

$$\frac{(\Delta x/x)_T}{(\Delta x/x)_L} = \frac{\lambda_o^2(\partial F/gMl)}{8F\lambda^3} \left(\frac{x}{l_o}\right)^2$$

$$= \frac{(\partial F/\partial\lambda)}{8F\lambda} \left(\frac{x}{l_o}\right)^2 \qquad \ldots 32$$

As an example, for rubber obeying the statistical theory stress-strain relation,

$$F = \frac{E_o'}{3} \left(\lambda - \frac{1}{\lambda^2}\right) \qquad \ldots 33$$

$$\partial F/\partial\lambda = \frac{E_o'}{3} \left(l + \frac{2}{\lambda^3}\right) \qquad \ldots 34$$

so *Equation 32* becomes

$$\frac{(\Delta x/x)_T}{(\Delta x/x)_L} = \alpha = \frac{(l + 2/\lambda^3)}{\delta\lambda(\lambda - \frac{l}{\lambda^2})} \left(\frac{x}{l_o}\right)^2 \ldots 35$$

$$\alpha = \beta \left(\frac{x}{l_o}\right)^2$$

Table 1 gives some examples of λ and β values.

TABLE 1. VALUES OF λ AND β

λ	β
1.1	1.040
1.2	0.445
1.5	0.1257
2.0	0.0446
3.0	0.0155
4.0	0.00818

Thus the damping present under transverse oscillations will be less than those for longitudinal oscillations by usually a large factor. For example, if $\lambda = 2$ and $x/l_o = 0.1$, α (the damping ratio) $= 4.46 \times 10^{-4}$.

Such a comparison should strictly be made at a transverse frequency of half that of the longitudinal frequency because each element in the strip undergoes two strain cycles for one complete oscillation in the transverse case, but only one in the longitudinal case.

EXPERIMENTAL

It has previously been observed during preliminary measurements that the transverse damping of oscillating stretched rubber strips, and in particular a gum rubber vulcanisate, can be extremely small. Thus, an important requirement of the test apparatus is to measure very low damping values quantitatively, or at least to show qualitatively, the large differences in damping that can occur between the two modes of oscillation.

To achieve as little inherent damping as possible, the apparatus was built using a beam supported by a knife edge, the arrangement is shown in *Figure 3,* assembled in the longitudinal oscillation configuration. A rubber test-piece is attached to the beam which is free to oscillate. The inertia of the oscillating beam can be adjusted to provide a decay in the oscillation amplitude suitable for the recording and measurement of each individual oscillation and associated frequency. Preliminary tests indicated that this system achieved a low damping support while providing sufficient oscillation amplitude for measurement purposes. Initially, a small groove was cut in the beam support plate to locate the knife edge, but this was found to significantly increase the friction at the contact point, and was unacceptable. It was found that the knife edge, in practice a razor blade, when supporting the beam dug into the beam support plate a sufficient amount to locate itself, and produced less damping than

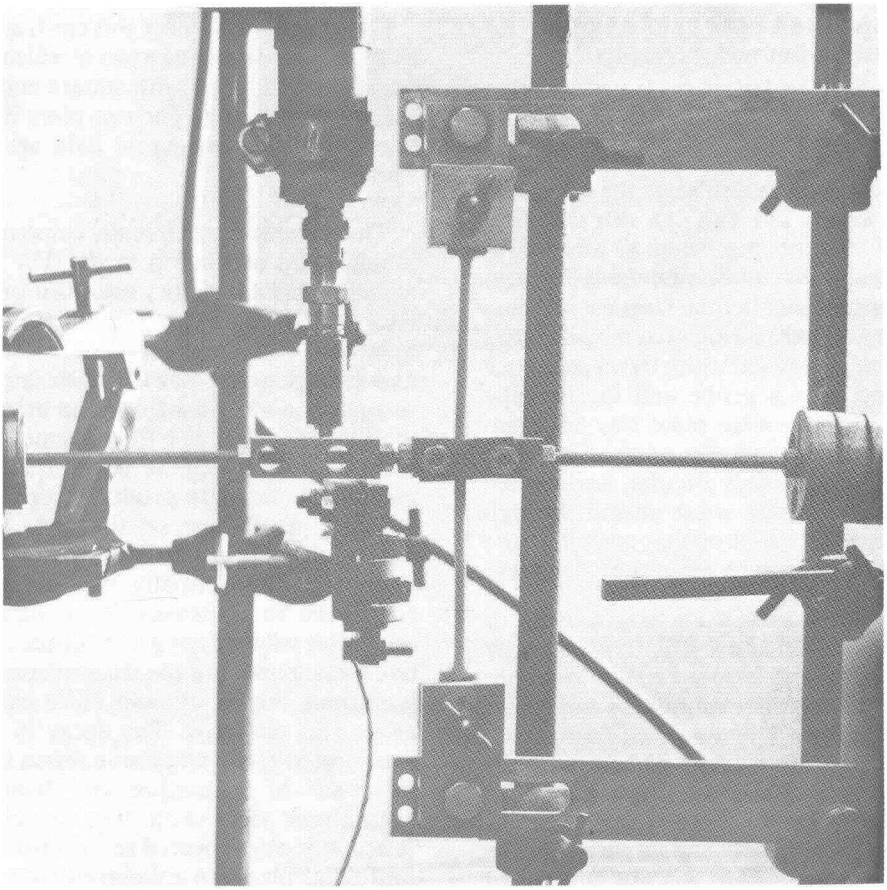

Figure 3. The apparatus constructed in the longitudinal oscillation configuration showing the beam supported on the knife edge with a test-piece in place and the displacement probe in position.

a deliberately cut locating groove. To achieve sufficient adjustment of the resulting oscillation frequency of the stretched rubber strips when fixed at the various test strains, a combination of weights could be added to the beam at any desired distance from the knife edge fulcrum.

The beam and knife edge supporting structure was constructed using a number of rods held together with screw clamps. This construction enabled ready adjustment of the test-piece strain and re-positioning of each test-piece in relation to the beam and probe. The rod and clamp construction also enabled the structure to be conveniently converted from the vertical test-piece configuration shown in *Figure 3*, for investigating longitudinal oscillations, to a horizontal test-piece configuration for transverse oscillation measurements.

In an attempt to reduce the inherent damping in the system by as much as possible, it was evident that the apparatus required to measure the frequency and amplitude of the test-piece oscillations could not rely on any physical contact with the moving beam as this would impose far too much additional damping, and would to some extent alter the frequency. Thus, a non-contacting measuring system was necessary and an arrangement using the capacitance developed between a probe and the metallic surface of the knife edge plate was adopted. Variations in the proximity of the probe to the flat metallic surface on the beam alter the capacitance, which when passed through suitable electrical equipment converts the capacitance changes into amplitude changes.

Further attempts were made to reduce the air damping present on the oscillating beam by shaping the beam and its attachments to reduce air friction while still maintaining sufficient inertia in the beam. The test-piece fixing plate and knife edge-beam support plate were made as small as possible and the beam itself constructed using a rod of relatively small circular cross-section. The inertia weights were in the form of thin discs. This shaping was most important with the weights, which being placed at the ends of the beam had the greatest distance to travel.

The complete apparatus is shown in schematic form in *Figure 4*. The two configurations for measuring the longitudinal and transverse oscillations were constructed in turn. The displacement between the probe and knife edge plate was recorded on the chart recorder *via* the probe electrical apparatus.

A fundamental point of interest is the degree of damping obtained in the transverse and longitudinal oscillating modes using a range of typical rubbers. Thus, measurements were conducted on three rubber compounds chosen to cover the spectrum of damping normally found in rubber-like materials. The vulcanisate details of the natural rubber based compounds used are given in *Table 2*.

Each rubber test-piece was cut from a rubber sheet by a die stamping process which produced parallel-sided strips with square end pieces to facilitate clamping. The test-piece dimensions together with stress-strain data are shown in *Table 3*.

The apparatus was initially constructed in the longitudinal oscillation mode. The beam and weights were carefully positioned on the knife edge so that the beam remained horizontally balanced with no test-piece attached. The capacitance probe was offset during the beam alignment and then adjusted to bring it within working range of the knife edge metallic pickup plate. Suitable weights were attached to the ends of the beam to produce an oscillation of 1 Hz. A gentle push set the beam in motion. Individual oscillations were recorded on the chart recorder. Initially measurements were conducted to determine the damping of the apparatus with no test-piece added. A quantitative measurement of the damping present in the apparatus due to air and knife edge friction alone was obtained. The decay in amplitude was plotted as log-amplitude *versus* the number of cycles of oscillation and from this the logarithmic decrement was calculated. The procedure was repeated several times with the knife edge placed in a slightly different position to determine any possible damping changes due to relocation but no measurable changes were detected.

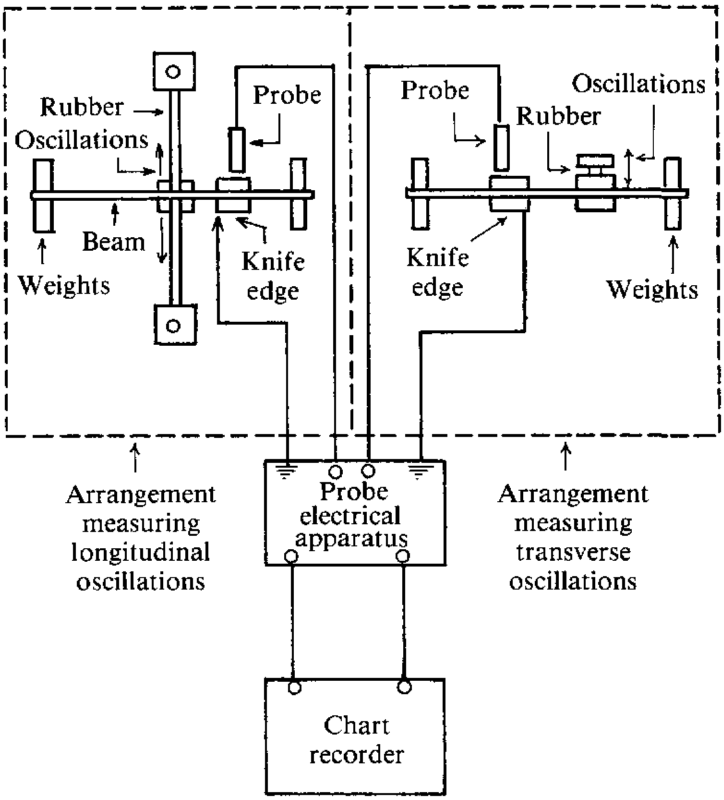

Figure 4. A schematic diagram of the complete apparatus showing the longitudinal and transverse oscillation configurations.

TABLE 2. VULCANISATE DETAILS OF NATURAL RUBBER BASED COMPOUNDS CHOSEN TO COVER THE SPECTRUM OF DAMPING

Item	Compound A	Compound B	Compound C
Natural rubber (SMR 5)	100	100	100
Zinc oxide	5	5	5
Stearic acid	2	2	2
HAF carbon black (N330)	—	45	—
Carbon black (N339)	—	—	80
Process oil	—	4.5	20
Santocure CBS	0.6	0.6	1.3
Sulphur	2.5	2.5	1.5
Vulcanisate cure	45 min 140°C	45 min 140°C	45 min 140°C
Vulcanisate hardness (IRHD)	43	64	68

TABLE 3. TEST-PIECE LOAD/EXTENSION DATA

Compound	Stress (MPa)	Strain (%)
A – Test-piece unstrained dimensions:	0.122	6.86
length 100 mm	0.245	15.69
width 3.82 mm	0.367	26.28
thickness 1.05 mm	0.489	39.61
C.S.A. 4.011 mm^2	0.611	55.71
	0.734	75.35
B – Test-piece unstrained dimensions:	0.101	1.177
length 100 mm	0.202	3.720
width 1.48 mm	0.303	6.851
thickness 1.31 mm	0.405	10.381
C.S.A. 1.939 mm^2	0.506	15.060
C Test-piece unstrained dimensions:	0.093	—
length 100 mm	0.155	0.339
width 1.47 mm	0.217	0.857
thickness 2.15 mm	0.279	1.614
C.S.A. 3.16 mm^2	0.341	2.600
	0.466	4.901

The influence of blade knife edge alignment was also investigated with a rubber test-piece in position, by deliberately mis-aligning the beam by a known amount. The procedure involved the use of a number of fixed test-piece strains and known amounts of mis-alignment. In each case, the frequency was found to alter by no more than 0.05 Hz and the log dec. value by a maximum of 5%. These results suggest that any mis-alignment occurring in practice will have a smaller effect than those of the deliberately mis-aligned arrangement and can be considered insignificant.

Having determined the damping present in the apparatus alone and the effects of any beam mis-alignment the various test-pieces were inserted in turn and the damping measurements undertaken. Each test-piece was clamped into position at the desired strain. Bench marks placed on each test-piece enabled the strain to be set, by sliding the test-piece clamps along the supporting rods and then tightening the clamps in position at the desired extension. The decay in amplitude of each oscillation and the frequency of oscillation were recorded at each test-piece strain and log-amplitude *versus* number of cycle plots drawn. After completion of all the longitudinal oscillation measurements, the apparatus was re-assembled in the transverse oscillation mode. The transverse oscillation measurements were now conducted using exactly the same procedure.

RESULTS

The longitudinal oscillation results are shown in *Figures 5, 6* and *7* for each of the three rubbers *(Compounds A, B, C)* used and are plotted as the logarithmic decay in amplitude *versus* the number of oscillations. *Figure 5* also includes the damping measurements conducted

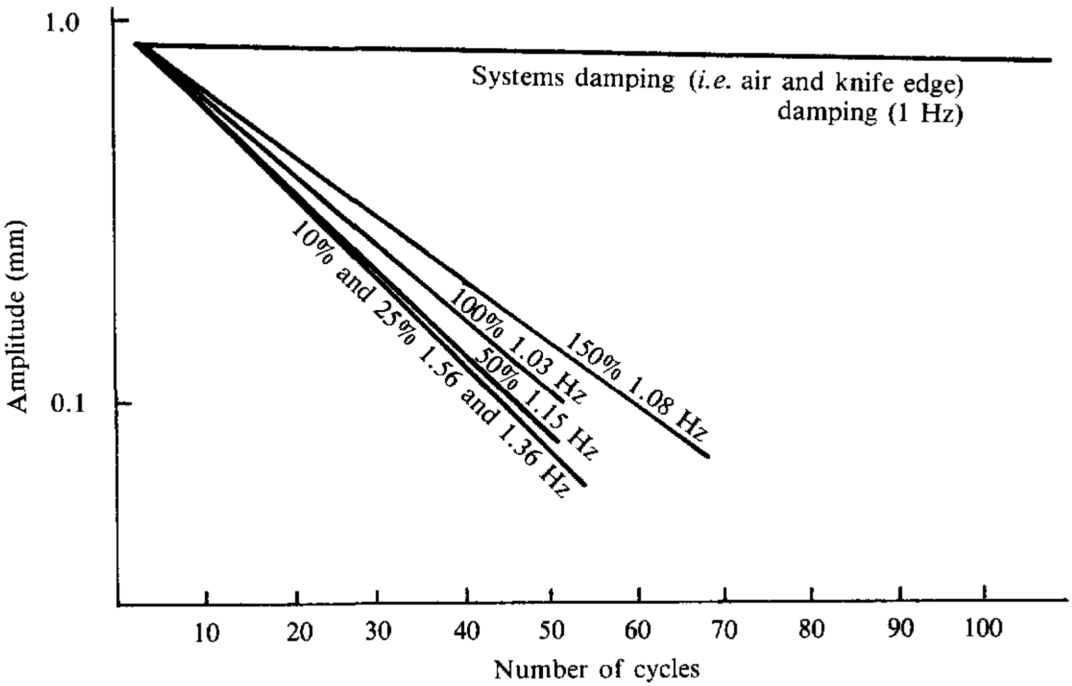

Figure 5. Oscillation amplitude versus number of oscillations of test-pieces (Compound A) fixed at strains between 10% and 150% at oscillation frequencies shown in the longitudinal oscillation mode. The damping present in the oscillating beam due to air and knife edge friction, is also shown at a frequency of 1 Hz.

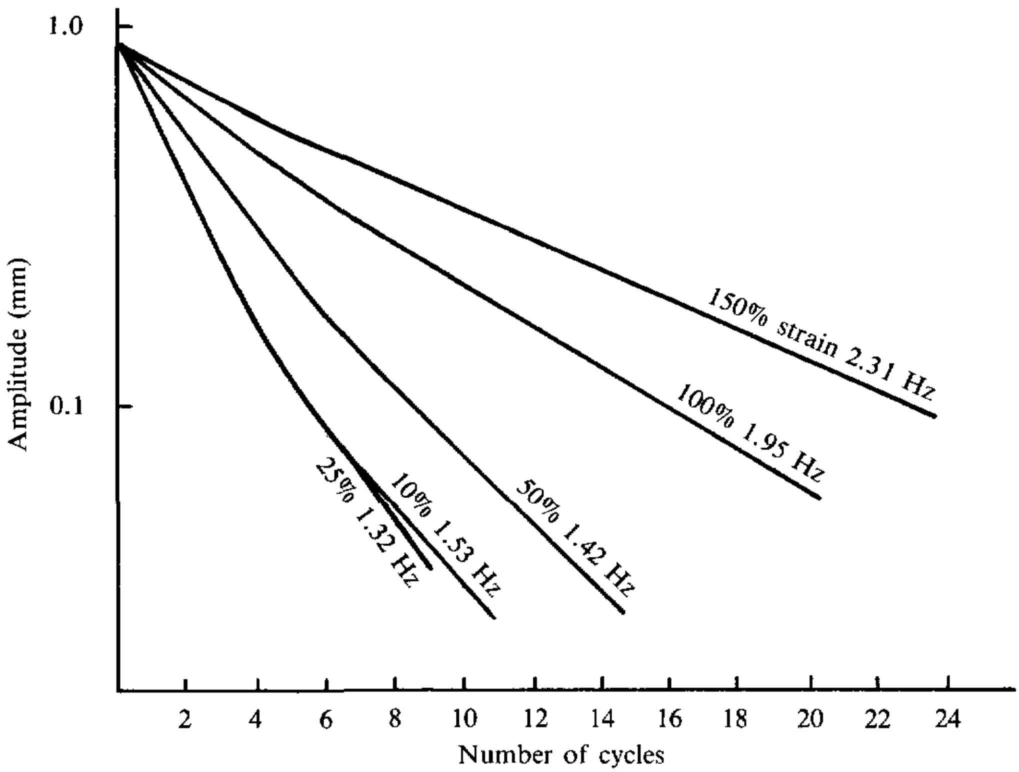

Figure 6. Oscillation amplitude versus number of oscillations of test-pieces (Compound B) fixed at strains between 10% and 150% in the longitudinal oscillation mode.

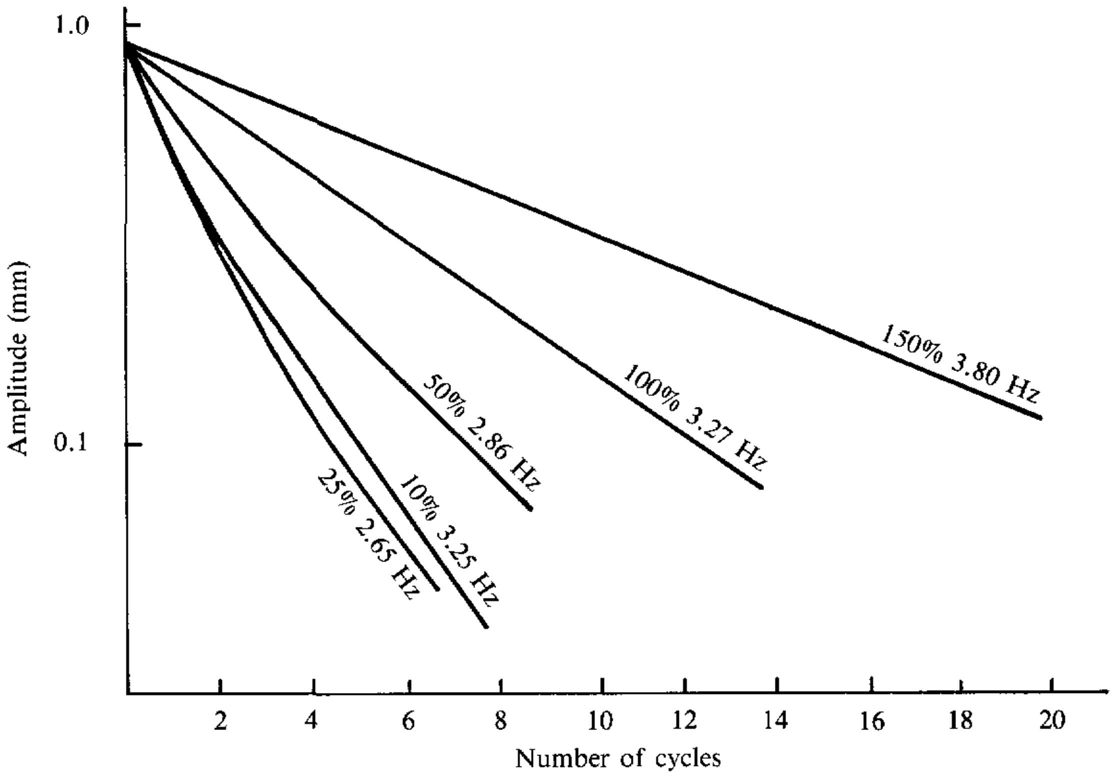

Figure 7. Oscillation amplitude versus number of oscillations of test-pieces (Compound C) fixed at strains between 10% and 150% in the longitudinal oscillation mode.

using the oscillating beam alone and without the inclusion of a rubber test-piece. This result provides a quantitative measurement of the damping present in the apparatus due to air and knife edge friction only. The transverse oscillation results are shown, plotted in a similar manner, in *Figures 8* and *9*. The change in damping as a function of test-piece tensile strain is given in *Figure 10*. The oscillation frequency and logarithmic decrement values obtained from these results at the various test-piece strains are shown in *Table 4*. Comparisons between the amount of damping obtained in the longitudinal and transverse oscillating modes for each rubber compound held at a similar fixed strain are also shown in ratio form in *Table 4*.

DISCUSSION

The large difference in damping qualitatively observed between a stretched rubber strip oscillating in longitudinal and transverse modes during preliminary experiments, has been quantitatively confirmed. The rubber damping in the transverse oscillation mode was found by experiment to be approximately 10 to 1400 times less than the damping in the longitudinal oscillation mode over the range of test conditions described.

Generally, the higher damping, carbon black-filled rubber vulcanisate *(Compound B)* when held at a small strain (10%) produced the least difference in damping between the two oscillation modes (approximately 10:1) while the lower damping gum natural rubber vulcanisate *(Compound A)* when held at a large strain (150%) produced the greatest difference in damping (approximately 1400:1). These differences are found to be in general agreement with values predicted by theory. However,

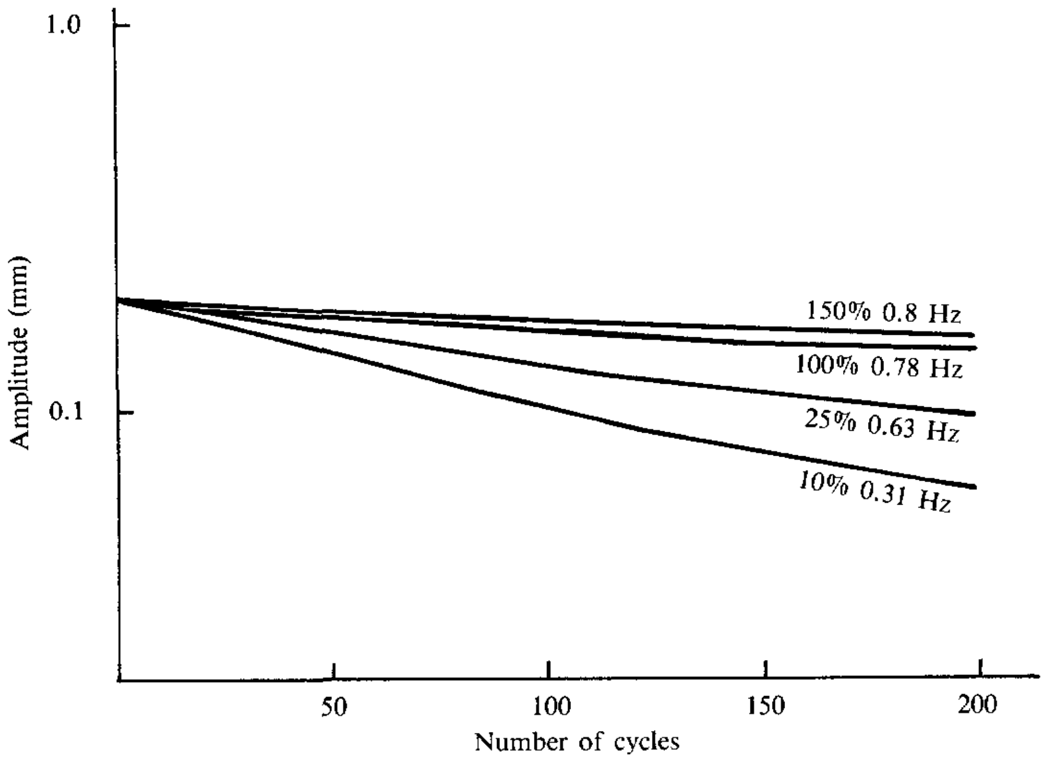

Figure 8. Oscillation amplitude versus number of oscillations of test-pieces (Compound A) fixed at strains between 10% and 150% in the transverse oscillation mode.

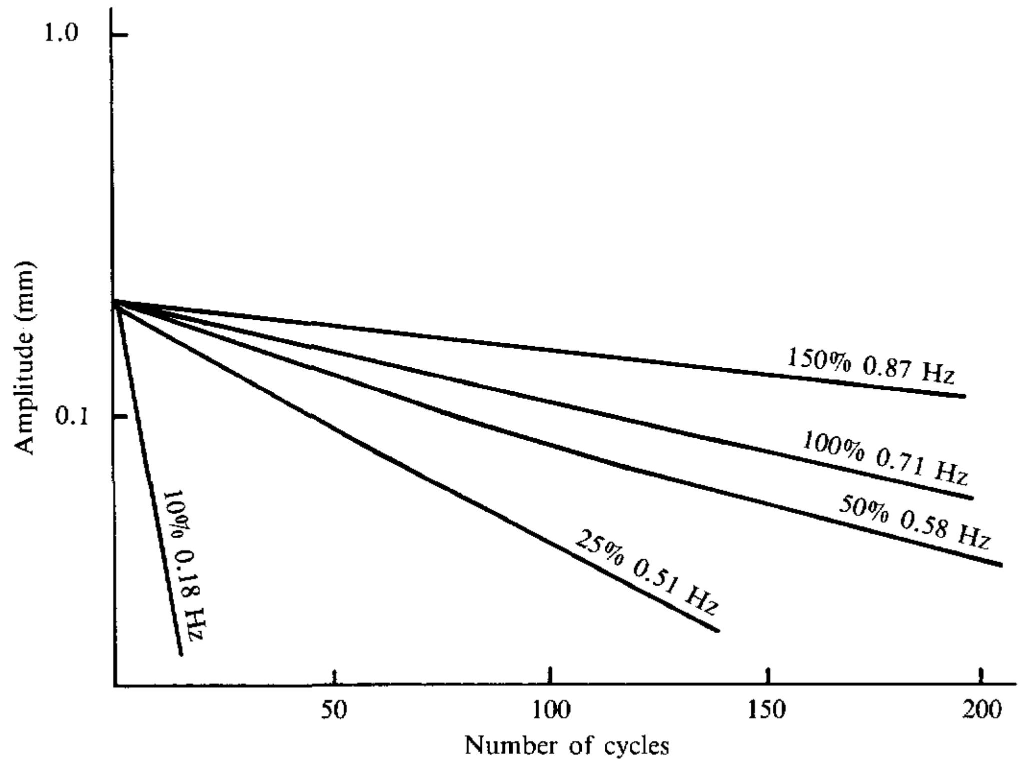

Figure 9. Oscillation amplitude versus number of oscillations of test-pieces (Compound B) fixed at strains between 10% and 150% in the transverse oscillation mode.

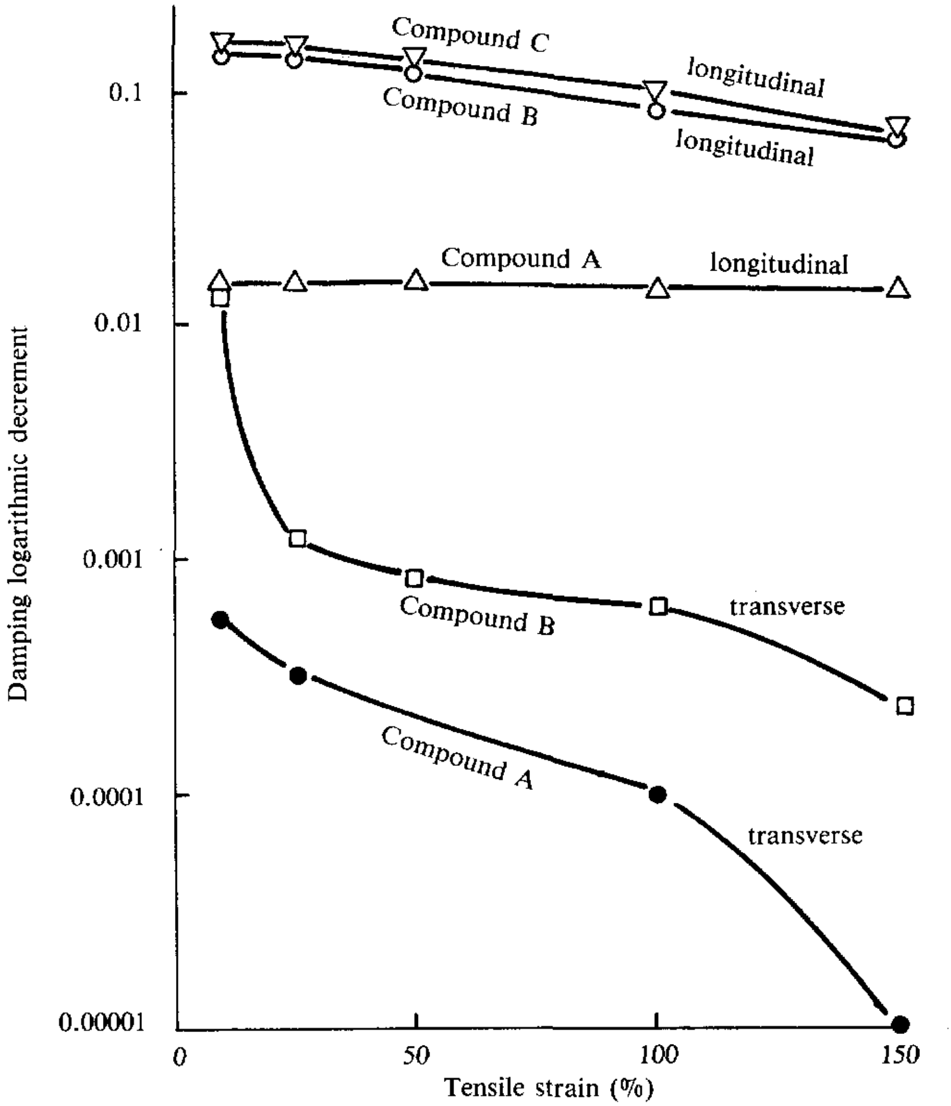

Figure 10. Change in damping as a function of tensile strain for transverse and longitudinal oscillations.

when very small amounts of damping were present, as is the case in the highly strained gum rubber vulcanisate oscillating transversely, other sources of energy loss due to air damping and clamping of the test-piece affected the quantitative agreement.

The large differences in damping observed between the two oscillation modes indicate the importance of the form a particular strain-energy cycle undergoes in determining the damping present in a strained vulcanisate and this will be particularly so for an unfilled or lightly-filled material where the effects of filler are not dominant.

In order to investigate the effect of strain on the damping behaviour of a stretch rubber strip, it is necessary to establish any effect on damping caused by a change in the oscillating frequency. The frequencies employed fell within the range 0.1 Hz to 4 Hz. This variation

TABLE 4. OSCILLATION FREQUENCY, LOGARITHMIC DECREMENT VALUES AND COMPARISONS OF AMOUNT OF DAMPING OBTAINED IN LONGITUDINAL AND TRANSVERSE MODES OF RUBBER COMPOUNDS AT VARIOUS TEST-PIECE STRAINS

Test mode	Test-piece vulcanisate	Test-piece strain (%)	Oscillation frequency (Hz)	Rubber damping, log/dec	Longitudinal to transverse rubber damping ratio (α)	Change in rubber damping, log/dec at 10% strain (%)
Longitudinal oscillations	Compound A	10	1.56	0.01559	28.3:1	0
		25	1.36	0.01561	47.3:1	+ 0.1
		50	1.15	0.01551		− 0.5
		100	1.03	0.01497	149.7:1	− 4.0
		150	1.08	0.01433	1433:1	− 8.0
	Compound B	10	1.53	0.14773	9.7:1	0
		25	1.32	0.14973	121.7:1	+ 1.4
		50	1.42	0.12773	153.9:1	− 13.5
		100	1.95	0.09013	143.1:1	− 40.0
		150	2.31	0.06773	282.2:1	− 54.0
	Compound C	10	3.25	0.15413		0
		25	2.65	0.15773		− 2.3
		50	2.86	0.13773		− 10.6
		100	3.27	0.10473		− 32.1
		150	3.80	0.06973		− 54.8
Transverse oscillations	Compound A	10	0.31	0.00055		0
		25	0.63	0.00033		− 40.0
		100	0.78	0.00010		− 82.0
		150	0.80	<0.00001		> − 98.2
	Compound B	10	0.18	0.01523		0
		25	0.51	0.00123		− 91.9
		50	0.58	0.00083		− 94.6
		100	0.71	0.00063		− 95.9
		150	0.87	0.00024		− 98.4

Apparatus damping with no rubber test-piece at 1 Hz (*i.e.* air and knife edge damping only) 0.00027

At oscillation amplitude (x) 0.2 mm

in frequency is the result of the increased restoring force present in a test-piece as the fixed applied strain is increased. It is known from previous work that the damping of a natural rubber gum vulcanisate *(i.e. Compound A)* is not strongly affected by a change in the oscillation frequency[1]. Over the working range of frequencies employed using *Compound A* (0.31 – 1.56 Hz), the damping will alter by only a small amount (approximately 2%). Considering, for example, *Compound A* oscillated longitudinally, a change in the applied strain of between 10% and 100%, results in a reduction in the damping log dec. value from 0.01559 to 0.01497 or a reduction in damping due to the strain alteration alone, of approximately 3.0%. This small reduction in damping caused by an increase in the strain was also found by Mason[2] when using frequencies of approximately 1 KHz. The effect of a change in strain on the transverse oscillation damping is, however, considerably greater. In this case, a similar strain change of between 10% and 100% produces a reduction in the log dec. value from 0.00055 to 0.0001, or a change in damping of at least a factor of five.

The carbon black-filled vulcanisates *(Compounds B* and *C)* both produced greater damping variations, due to changes in the test strain, when compared to the gum vulcanisate. The presence of non-elastic carbon black particles in a vulcanisate means the remaining rubbery material in a test-piece undergoes a somewhat larger extension for any given fixed strain than an unfilled vulcanisate. This effect is known as strain amplification. The effect of strain amplification on *Compound B* is to increase the average strain in the rubber phase by approximately 30% over that of the gum vulcanisate. Since the effect on damping of such a strain change is small when oscillated longitudinally as shown by the results using *Vulcanisate A*, the majority of the difference in damping observed between the gum and carbon black-filled vulcanisates can be associated with the introduction of the carbon black filler itself. Thus, a major factor in determining the quantity of damping present can be associated with the filler-filler and rubber-filler interactions taking place in the oscillating strained state. In the longitudinal oscillation mode, for example, the damping in *Compound B* is increased by approximately 60% when the strain is reduced from 100% to 10% by these filler effects. In the case of the transverse oscillations, the effects of the filler on the damping are greater, here a strain reduction from 100% to 10% produces an increase in damping of approximately twenty-seven times. It is believed that the rubber-filler interaction involves separation between the carbon particles and the bulk of the rubber. Carbon particles will also make physical contact between themselves in some instances and such contact will be a further source of damping.

The test apparatus has proved sufficiently sensitive and precise to measure levels of damping normally experienced throughout the range of rubbery materials and has proved particularly useful where small amounts of damping are present, as in most gum rubber vulcanisates where such measurements using dynamic testing machines can be difficult. Since the test-piece oscillation frequency is influenced by the inertia of the oscillating beam and the inertia in the beam can be altered by applying suitable weights a considerable range of frequencies is in principle possible using this test apparatus. The present operational frequency range is limited to below approximately 10 Hz only by the response of the chart recorder, by using a high speed recording device such as a transient recorder considerably higher frequencies could be employed. The test apparatus could be made considerably smaller and more portable, and would be a useful and inexpensive piece of apparatus for general laboratory use.

CONCLUSIONS

The large differences in damping qualitatively observed between a stretched rubber strip oscillating in longitudinal and transverse modes during preliminary experiments, have been studied quantitatively in the experiments described here, and found to be in general agreement with damping values predicted by theory. The damping present in a vulcanisate is shown to be strongly influenced by the tensile strain when oscillated transversely but much less so when oscillated longitudinally.

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