Rubber Friction: 'Wet' Schallamach Waves

A.D. ROBERTS*

For dry sliding contact between rubber and a hard counterface, Schallamach showed that relative motion was sometimes only due to 'waves of detachment' crossing the contact zone. It now appears that these waves propagate for certain conditions of water lubricated contact. Described here is a study of the wet waves which includes the conditions for their appearance, the observed level of friction and some analysis in terms of surface energy.

Schallamach waves of detachment¹ at the sliding interface of dry rubber surfaces have been the subject of considerable study with a view to elucidating the friction mechanism²⁻⁶. Experiments over the years have shown the importance of surface adhesion in this mechanism⁷⁻¹⁷. Since their discovery in 1970, the waves have been described extensively for dry contact between optically smooth, solvent-cleaned surfaces, suggesting that perhaps these somewhat artificial conditions were a pre-requisite for the formation of waves upon the application of tangential stress.

The chance discovery now has been made that Schallamach waves propagate in *water*lubricated contacts of smooth rubber sliding on glass. In view of earlier success based on the surface energy approach to explain the dry waves, theories along similar lines are being formulated for the wet waves to help in the prediction of wet friction. The discovery indicates that Schallamach waves, far from being just a 'laboratory phenomenon', may occur more widely in practice than hitherto suspected. This should lead to a fuller understanding of the behaviour of rubber on the wet contaminated surfaces which are encountered, for example, on roadways.

Studies of the wet waves are reported here in detail. These include the conditions for their appearance, how they were observed experimentally and analysed theoretically. This then indicates how the level of wet friction can be predicted in the presence of the wet waves, knowledge of which may find practical application.

THEORETICAL BACKGROUND

Schallamach observed¹ that the relative motion between dry, soft rubber and a hard track was often only due to clearly visible folds in the rubber surface that flickered across the contact region. Between the folds, there appeared to be solid contact. Subsequent observations of these 'detachment waves' revealed, under the microscope, some of the conditions for their appearance in dry contact². Later studies of the sliding friction of rubber^{3,4} support a mechanism in which energy is lost by the peeling of rubber at the interface of a wave⁵. It is produced when the tractional force creates a compressive stress in the edge of the contact region, causing the rubber to buckle through elastic instability^{1,2,6,7}. For the wave to propagate across the contact region there must be continuous unpeeling from and re-adhering to the hard track. To unpeel soft smooth rubber from glass requires considerable energy^{5,8}, the process being much influenced by viscoelasticity^{9,10}. Evidence¹¹ suggests that the apparent peel energy required to separate surfaces is of the form $\gamma_P = \gamma_O f$ (hysteresis), where γ_O is the equilibrium energy associated with the change in interfacial area and where the function of hysteresis may be up to 10^4 in magnitude. Thus the question arose as to whether the peeling of rubber away from the track was the main origin of the frictional work

^{*}Malaysian Rubber Producers' Research Association, Brickendonbury, Hertford, SG13 8NL, United Kingdom

that is done in sliding a smooth surfaced sample, rather than bulk deformation losses. Experiments for a range of rubber vulcanisates^{3,4,14} demonstrate that some 50%–90% of energy dissipation can be accounted for by wave peeling.

From these studies, it appeared that dry contact was necessary for sufficient interface adhesion to under-pin the formation of the waves upon application of tangential stress. Thus, the appearance of 'wet' waves came as a surprise.

In addition to dry surfaces, the various studies suggest that for the appearance of waves, sliding surfaces should be very smooth, the rubber of low modulus and the relative sliding speed at an optimum of a few tenths of a millimetre per second. Not surprisingly, these special conditions might suggest that the waves are just a laboratory phenomenon.

Earlier it was shown^{3,5} for rubber in dry sliding contact with glass how the tangential stress F might be related to the rate of wave propagation according to the relation:

$$F = \gamma_{p} w / \lambda V \qquad \dots 1$$

where γ_P is the relevant interfacial peel energy, $\gamma_P = \gamma_P$ w the wave velocity (which dictates the value of γ_p), λ the wave spacing and V the body sliding velocity. This relationship assumes all energy to be dissipated in the wave peel process at the interface (but see also Reference 14). It may be noted that the adhering energy, γ_{a} , is small, thus the losses are related to $\gamma_{\rm p}$ to a good approximation, and not $\gamma_p = \gamma_a$. The peel energy for dry contact depends upon the surface energies of the individual halves of the interface and the visco-elasticity of the rubber. After Gent and Schultz¹¹, a relationship might be written as:

$$\gamma_P = (\gamma_{RA} + \gamma_{GA} - \gamma_{RG}) f_H \qquad \dots 2$$

where suffixes are R for rubber, G for glass, A for air and H for hysteresis (in the rubber). The rate-dependent visco-elastic factor, f_H , is characteristic of the particular rubber vulcanisate under test. For wet contact this expression becomes:

$$\gamma_P^{-1} = (\gamma_{RW} + {}_{GW} - \gamma_{RWG}) f_H^1 \qquad \dots 3$$

where W is for water. The visco-elastic factor, f_{μ}^{1} is written differently to that in Equation 2 to allow for any water within the rubber altering hysteretic properties, and to recognise that the factor may not be independent of the surface energy. At the present stage of development, both γ_p and γ_p^{-1} need to be determined experimentally (at the appropriate wave velocity) before any friction prediction can be made from *Equation 1*. In what follows, all contacts may be considered as solid-liquid-solid.

EXPERIMENTAL

Wet Contact Area Observations

The experiment performed to show the generation of wet Schallamach waves was as follows. A glass plate, which was a prism beamsplitter designed for viewing the contact region in high optical contrast¹⁸, was loaded against a smooth rubber hemisphere that had been lubricated with a few drops of water (Figure 1). The contact area between the two surfaces could be viewed and photographed through a low power microscope. Under load, the trapped film of water thinned upon normal approach of surfaces, so leading to close approach (appears black) between rubber and glass over the entire geometric contact region but with a thick film of water remaining at the contact periphery. The manner of the contact region water film collapse has been described in earlier articles^{18,19}. If then the rubber was slowly displaced tangentially to the glass, Schallamach waves generally could be seen moving quickly in the displacement direction through the visibly black region of contact. Sometimes the waves were only apparent as flashes of light. Upon careful inspection, it became clear that the waves were transporting water through the contact region, just as in dry contact they transport air. In the wake of each wet wave, the rubber apparently seals back onto the glass, though less firmly with each passing wave. If surfaces were rapidly displaced tangentially, it was difficult to see the start-up waves for the rubber soon de-adhered and elastohydro-

RL



Figure 1. Apparatus for observations of 'wet' waves. The progress of contact events is watched in white light (relatively dim) and photographs taken by flash light. The microscope stage traverse is driven by a variable speed electric motor. Friction between rubber and glass surfaces is measured by displacement of the glass plate held on a calibrated spring support. Motion is perpendicular to the plane of this diagram.

dynamic sliding set in over the entire contact region with the appearance of Newton's colours in the lubricating water film.

Photographic Technique

The wet waves can be more clearly identified and examined by flash photography using effective exposure times of 1/1000 s or less.

Schallamach waves were first observed for *dry* contact. While the wet waves can be seen fairly easily by direct viewing through the low power microscope *(Figure 1)*, to photograph them presented technical difficulties. The presence of water much diminishes the optical contrast. Further, the reflected light intensity of a water contact image is an order of magnitude less than for dry contact. Flash lamps were placed near to the prism beamsplitter for maximum light input, but even so it was necessary to use the fastest available Polaroid film (3000 ASA) which unfortunately results in some graininess on photographs.

By using two flash lamps, it became possible to take double or multiple exposure photo-

graphs in order to examine the progress of a wave across the contact region, or to record contact periphery movements. The disposition of components to make multiple flash exposures is shown (Figure 1). The contact region was continuously viewed in white light. To take photographs, the flash lamps were brought into operation through an electrical contact with the microscope camera shutter and a mechanical delay system to put a variable time interval (1/100 to 1 s) between each flash. The mechanical delay was effected through a double contact on a spinning wheel driven by a variable speed electric motor. A 45/55 reflection/ transmission mirror is placed before the flash lamps. The result of using this mirror is that light flashes arriving at the contact region are of different intensity, so that it is possible, on a black and white photograph, to distinguish one flash from the other.

Two distinct advantages follow from the use of flash lamps. Firstly, effective exposure times are determined by the flash lamp, and may be between 1/1000 and 1/20 000 of a second, depending upon the setting of the flash lamp computer. Thus rapid movements can be recorded without blur. Secondly, it is more convenient for comparison and measurement to have two exposures on one photograph. If high speed cine film were to have been used, adjacent frames would have had to be compared by physical overlap or other involved means.

Friction Measurements

The sliding friction between rubber and glass was measured by the tangential displacement of a spring support attached to the glass plate. The displacement was measured optically through a travelling microscope. The spring was fairly rigid in order to support the plate firmly, so displacements were small but could be easily followed through the microscope. Tangential displacements were calibrated using weights connected by thread over a pulley wheel to the plate. The traverse was driven by a small electric motor *via* a reduction gearbox, to provide several decades in sliding speed.

For more extensive friction measurements (but no photographs), optically transparent rubber hemispheres were run against a lubricated glass turntable (*Figure 2*). The interface was viewed through the rubber while the friction was measured by a load cell. An electric drive *via* a gearbox to the turntable gave a range of speeds from 10^{-3} to 10^{3} mms⁻¹. Normal loads were applied by dead weight on the lever arm, contact pressures being in the range 0.1 MPa to 1 MPa.

The rubber samples used were compression moulded hemispheres possessing an optically smooth surface²⁰. Their diameter was 37 mm and compound details are given in *Table 1*. Vulcanised samples were cleaned with acetone, as necessary. The counterface material was soda glass, also solvent cleaned with acetone and finished with pure isopropyl alcohol. Water used (as lubricant) was simply of once-distilled quality.

RESULTS

Start-up Wet Waves

With rubber and glass only separated by a few monolayers of water (as per experimental

description above), surfaces were set into slow tangential displacement under a modest normal load. Photographs of the start-up sequence were shown (*Figure 3*). The friction rises with increasing displacement, the 'black' region in which wet waves propagate becoming smaller. Around this region a film of water exists, the thickness of which is indicated by the interference fringes.

The level of friction is determined mainly by the central 'black' contact region over which the wet waves peel. The waves transport water through this region, curiously enough from the back to the front of the contact. Estimates of the friction magnitude can be made from observations on the waves through the use of Equation 1 and writing γ_p^{\perp} for γ_p (see Equation 3). The photographs shown (Figure 3) are double-flash exposures. From the first (or second) exposure the wave spacing, λ , can be found. Knowing the time lapse between the two exposures enables the wave speed, w, to be determined.

Wave Peel Energy

In the equation, γ_{p}^{-1} is the peel energy at the corresponding rate for wet contact, and is determined independently from a wet peeling test (Figure 4). In this test, a rubber hemisphere is loaded against a wet glass plate and the water film allowed to collapse, as described above. The applied load is then removed (reduced to zero), whereupon the contact periphery rapidly moves 'inwards' to a smaller circle of contact. This peeling event is photographed by multipleflash exposures (Figure 4). From such photographs, the interfacial peel energy in the presence of water can be determined at any contact diameter by the relationship²¹ between adhesion energy and contact geometry according to

$$\gamma_P^{-1} = \frac{8 E a^3}{27 \pi R^2} \qquad \dots 4$$

Changes in contact diameter with time (flash intervals) allow the peel rate to be calculated, so that a plot of γ_{p^1} with peel rate can be drawn (*Figure 5*) in order to select the appro-



Figure 2. Apparatus for measuring continuous sliding lubricated friction. The contact region is viewed through the transparent rubber hemisphere as the glass turntable rotates. Oblique illumination through a roof-shaped 'Perspex' prism allowed the contact region to be seen in good optical contrast.

Compound	Formulation (parts by weight)		
	A	В	
Natural rubber (SMR-L)	100		
Polyisoprene (Cariflex 305)		100	
Zinc oxide	5		
Stearic acid	2		
Nonox ZA	1		
Sulphur	2.5		
Santocure NS	0.5		
Dicumyl peroxide		2	
Cure time/temp.	40 min/140°C	50 min/160°C	
Hardness (IRHD)	44	40	



(a) Friction force 0.027N



(b) Friction force 0.035N



(c) Friction force 0.037N



(d) Friction force 0.038N

Figure 3. Start-up sequence of contact area events. As the displacement of the rubber hemisphere (from left to right) increases, the wet waves appear. With fully established sliding (d) the waves fragment. In these double flash photographs, the waves are seen moving from left to right through the central 'black' contact region. Around the region there is water and the contour of the rubber surface can be found from the surrounding interference fringes. The sliding speed is 0.05 mms⁻¹ and the major elliptical contact diameter is approximately 2.2 mm (varies slightly from photograph to photograph). The normal load was 0.05 N and the friction force rose with increasing displacement.



Figure 4. Contact area wet peeling examined by multiple flash photography. This case shows three stages of peeling in the presence of water. The initial outer diameter of contact is 2.5 mm, achieved under an applied load of 0.49 N for a dwell time of 0.5 h. Upon reducing the applied load to zero, the contact diameter shrinks to 1.6 mm approximately in 0.4 s, so giving an average peel rate of 9×10^{-2} mms⁻¹. The corresponding average peel energy is 0.8 Jm⁻². Further shrinking to a diameter of 1.3 mm approximately took 30 s, giving corresponding rate and energy values of 4×10^{-3} mms⁻¹ and 0.17 Jm⁻². In practice, it was possible to take up to seven flash exposures on a single 3000 ASA Polaroid print, which gives six pairs of rate/energy data points.

priate value of γ_p^{-1} for insertion into Equation 1 for predicting the friction. The appropriate value is that for the peel rate as observed during wave propagation. The peel results (Figure 5) illustrate the different energy levels according to whether the contact was dry, wet and wet with some surface contamination.

Theory and Experiment

A comparison of predicted and actual friction values is shown in *Figure 6*. It suggests that up to some 60%-80% of the friction observed can be accounted for by peel energy dissipation, during the generation of wet waves.

Continuous Sliding

Friction measurements were made using a transparent rubber hemisphere (Compound B) sliding on a water-lubricated glass turntable

over a range of speeds. The contact zone was viewed through the rubber.

At low sliding speeds, less than 0.1 mms^{-1} . 'wet' Schallamach waves could be clearly seen in the contact zone. With speed increase, 0.1 mms⁻¹ to 10 mms⁻¹, the waves appeared more like undulating ripples in the rubber interface, sometimes giving the impression that the contact region was 'twitching'. Presumably this is an intermediate situation between boundary and elastohydrodynamic lubrication, sometimes referred to as the regime of mixed lubrication. At higher speeds, greater than 10 mms⁻¹, uniform steady sliding was obtained with a water film separating rubber from glass. The thickness of the film could be judged by its Newton's colours. For example, at a speed of 100 mms⁻¹, the water film appeared yellowblue. Suitable calibration (Newton's rings in white light for the contact glass plate-water-



Figure 5. Variation in peel energy with peel rate for dry and wet contact of rubber against glass. Wet contact dwell times were 0.5 h. Wet energy levels were noticeably sensitive to surface contamination such as dust or bloom. They were maintained in a good state by solvent cleaning with 10% acetyl-acetone (AA) in isopropanol. Whether wet or dry, energies increase with peel rate.

rubber sphere) allowed interpretation of such colours into a film thickness, this particular case corresponding to a thickness of 0.35 μ m.

The overall pattern of water-lubricated friction results are shown (*Figure 7*) for different speeds. The regimes of contact behaviour are indicated. The hydrodynamic shear resistance was calculated at each decade of speed using the Newton formula $f/A = \eta(v/h)$, where f is the shear resistance over the contact area A. In these experiments the viscosity of water may be taken as $\eta = 10^{-3}$ Pas. The film thickness h can be calculated or directly measured. Estimates of the minimum film thickness²² may be made according to $h_{\min} = 1.5(\eta u)^{0.57} R^{0.56} (w/l)^{-0.13} (8E/3)^{-0.44} \dots 5$ where R is the rubber sphere radius, E its Young's modulus, W the applied load and I the contact diameter. The surface velocity is given by $2u = v_1 + v_2$, where v_1 is the glass turntable speed and v_2 that of the rubber sphere (here $v_2 = 0$). When a fully developed water film is formed, theoretical estimates and experimental results are in reasonable agreement (Table 2). However, the calculated hydrodynamic shear resistance, $\pi a^2 \eta v/h$, did not agree with the measured friction force. The comparison (Table 2) implies that only at high sliding speeds is there a complete water film. At low speeds, the high friction indicates much solid-solid interaction between sliding surfaces.



Figure 6. Comparison of predicted and actual friction force values at the initiation of sliding. The contact diameter is 2a. The data shown were derived from such photographs as those shown in Figure 3. The error bars on calculated friction values indicate limits arising out of variation in wave speed and spacing across the photographed contact areas. The applied load was 0.49 N, the initial dwell 0.5 h and the subsequent sliding speed 0.05 mms⁻¹. The vulcanised rubber hemisphere was made from Compound A (Tempt = 20° C).

Waves in Presence of Silicone Fluid

An investigation was made to see whether Schallamach waves would arise in the presence of a silicone fluid as a lubricant for the sliding of a rubber hemisphere against a glass plate. A natural rubber vulcanisate (Compound A, Table 1) was lubricated with a dimethyl silicone of viscosity 12.5 Pa s⁻¹. Simultaneous observations were made of the friction and contact area.

Rubber and glass surfaces were loaded together in the presence of the silicone fluid and

a long dwell time allowed (five days) for the fluid to be expressed from the contact zone. The contact was then slid. Fragmentary waves were observed during the start-up sequence (Figure 8) if the sliding speed was relatively low (0.05 mms⁻¹). Wave speeds were of similar magnitude to the sliding speed. Estimates for the wave separation were 0.7 mm and, by independent experiment giving results similar to Figures 4 and 5, for the interfacial peel energy 125 mJm⁻², so that the calculated friction force (from Equation 1) was of the order 10^{-4} N. Measured frictional forces were con-



Figure 7. Coefficients of friction at different speeds for a rubber hemisphere sliding on a water flooded glass turntable. The regimes of lubrication behaviour are indicated (Temp. = 20° C). Both the rising 'dryish' friction and the falling lubricated may be associated with velocity increase, the former reflecting visco-elastic dissipation and the latter hydroplaning.

Sliding speed (mms ⁻¹)	Observed film thickness (µm)	Theoretical film thickness (µm)	Hydrodynamic shear resistance (mN)	Measured friction force (mN)
0.001	zero	0.0005	0.04	990
0.01	zero	0.0017	0.11	1 000
0.1	0.02	0.0065	0.28	1 540
1.0	0.05	0.024	0.76	880
10	0.10	0.088	2.1	390
100	0.35	0.33	5.5	40
200	0.55	0.49	7.5	< 40
1000	> 1	1.24	14.5	< 40

TABLE 2. WATER LUBRICATION OF RUBBER SPHERE SLIDING ON GLASS

Rubber hemisphere = Compound B; Load 2.2N; Temp. $\approx 17^{\circ}C \pm 2^{\circ}C$

siderably greater (1 to 6×10^{-2} N), which may imply extensive solid-solid surface interactions, although in the case of a viscous fluid the hydrodynamic shear resistance will be significant. Suppose fluid films are of the order of 100 nm (quarter wavelengths, whitish hue), then the hydrodynamic resistance would be about 2×10^{-2} N.



Figure 8. Production of fragmentary Schallamach waves in the presence of silicone fluid. The contact diameter is 2.3 mm. Sliding speed 0.05 mms⁻¹, silicone viscosity 12.5 Pa s⁻¹, normal load 0.03 N, friction coefficient 0.2 (Temp. = 20° C).

DISCUSSION

It would appear that for the often encountered practical situation of wet contact, Schallamach waves may arise at areas of close approach. Visco-elasticity of the rubber surface is known to enter into the peel process as waves propagate and provide a mechanism for energy loss (ultimately as heat in the rubber). The equilibrium interfacial surface energy γ_{0}^{1} for contact under water is less than for dry contact so the product $\gamma_{0}^{1} f$ (hysteresis) will be less and therefore tractional resistance less than when dry. While some energy may be lost in the hydroplane films of water adjacent to the regions of contact, more may be lost where Schallamach waves propagate. In studies of skidding wet road tyres, footprint photographs often suggest that the contact consists of three zones hydrodynamic, partial lubricated and 'dry' contact²³. It has been suggested that most resistance to sliding is acquired in the 'dry' zone of tyres. Once the 'dry' footprint region of the

tyre is lost at high speed due to hydroplaning, then effective skid resistance is lost.

The so-called regions of 'dry' contact cannot be considered as regions of genuine dry contact. Liquid within a wet wave is being propagated through such regions and associated surface energies are much less than for dry contact *(see Figure 5)*. In static contact, any trapped liquid will try to escape¹⁸. However, all the observations (wet waves and squeeze films) imply the presence of residual liquid monolayers between the so-called 'dry' contact surfaces. Hence, the practical significance of the 'wet' Schallamach wave observations.

The interfacial surface energy for intimate contact in the presence of organic fluids is likely to be less than for water. If such fluids are viscous, then the hydrodynamic shear resistance may become significant. Preliminary tests presented here for a viscous silicone fluid suggest that a greater proportion of the measured traction derives from the hydrodynamic shear resistance rather than from the 'dry' contact zone.

We note that in a laboratory report²⁴ published some years ago, there was an observation suggesting the occurrence of Schallamach waves in the presence of mineral oil for the contact of soft silicone rubber with glass. The authors of the report, however, rejected the idea of Schallamach waves in this context because it was in the presence of an organic liquid, and, the waves were not moving with their characteristic high speed. In view of the present results, it may not be unreasonable to suppose that these authors were observing Schallamach waves in liquid contact. Their slow speed can be reconciled when it is recalled that the wave speed will be determined by the materials in the contact under test. The particular silicone rubber used was different from the NR vulcanisate originally used by Schallamach.

At a seminar on rubber friction some years ago, a comment was made to the effect that Schallamach waves had been seen in the presence of the liquid perfluorodecane sandwiched between silicone rubber and glass²⁵. These two additional observations add to the picture which suggests that Schallamach waves are a rather more universal phenomenon than at first thought. If this is the case, it becomes much easier to see how visco-elasticity, well known in general for its influence upon rubber friction, enters into the wet skid process at the interface.

CONCLUSION

The discovery that Schallamach waves, first observed for the dry sliding contact of rubber, can be seen even in *water*-lubricated contact is reported. It suggests that the waves may occur more widely in practice than hitherto suspected. The present experiments indicate how the level of friction for a particular rubber vulcanisate can be predicted approximately in the presence of the 'wet' waves. With sliding speed increase, the waves fragment and disappear with the onset of full fluid film lubrication. It is only in the so-called 'mixed regime' of lubrication that the 'wet' waves are likely to occur.

ACKNOWLEDGEMENT

The author has pleasure in thanking Dr A. Schallamach and Dr K.N.G. Fuller for helpful comments on the manuscript.

Date of receipt: December 1988 Date of acceptance: April 1989

REFERENCES

- 1. SCHALLAMACH, A. (1971) How does Rubber slide? Wear, 17, 301.
- BARQUINS, M. AND COURTEL, R. (1975) Rubber Friction and the Rheology of Viscoelastic Contact. Wear, 32, 133.
- 3. ROBERTS, A.D. AND JACKSON, S.A. (1975) Sliding Friction of Rubber. *Nature*, 257, 118.
- BRIGGS, G.A.D. AND BRISCOE, B.J. (1975) Dissipation of Energy in Friction of Rubber. Wear, 35, 357.
- 5. ROBERTS, A.D. AND THOMAS, A.G. (1975) The Adhesion and Friction of Smooth Rubber Surfaces. *Wear*, 33, 45.
- GENT, A.N. (1974) Friction and Wear of Highlyelastic Solids. Wear, 29, 111.
- ROBERTS, A.D. AND THOMAS, A.G. (1976) Static Friction of Smooth Clean Vulcanized Rubber. NR Technol., 7, 38.
- KENDALL, K. (1974) Effect of Relaxation Properties on the Adhesion of Rubber. J. Polym. Sci., A2(12), 295.
- GENT, A.N. AND PETRICH, R.P. (1969) Adhesion of Viscoelastic Materials to Rigid Substrates. Proc. R. Soc., A310, 433.
- ANDREWS, E.H. AND KINLOCH, A.J. (1973) Mechanics of Adhesive Failure. Proc. R. Soc., A332, 385.
- GENT, A.N. AND SCHULTZ, J. (1972) Equilibrium and Non-equilibrium Aspects of the Strength of Adhesion of Viscoelastic Materials. Proc. Int. Rubb. Conf. I.R.I. London.
- BRIGGS, G.A.D. AND BRISCOE, B.J. (1978) How Rubber Grips and Slips. *Philos. Mag.*, A38, 387.
- BEST, B., MEIJERS, P. AND SAVKOOR, A.R. (1981) The Formation of Schllamach Waves. Wear, 65, 385.
- BARQUINS, M. (1983) Energy Dissipation in Schallamach Waves. Wear, 91, 103.

- BARQUINS, M. AND ROBERTS, A.D. (1986) Rubber Friction Variation with Rate and Temperature. J. Phys. D: Appl. Phys., 19, 547.
- BARQUINS, M. AND ROBERTS, A.D. (1987) Adhesion and Friction of Epoxidized Natural Rubber Vulcanizates. J. chem. Phys., 84, 225.
- ARNOLD, S.P., ROBERTS, A.D. AND TAYLOR, A.D. (1987) Rubber Friction Dependence on Roughness and Surface Energy. J.nat. Rubb. Res., 2(1), 1.
- ROBERTS, A.D. (1971) Squeeze Films between Rubber and Glass. J.Phys.D:Appl.Phys., 4, 423.
- ROBERTS, A.D. (1977) Studies of Lubricated Rubber Friction. *Tribology Int.*, 10, 115.
- ROBERTS, A.D. (1976) Optical Rubber. Rubb. Dev., 29, 7.

- JOHNSON, K.L., KENDALL, K. AND ROBERTS, A.D. (1971) Surface Energy and the Contact of Elastic Solids. Proc. R. Soc., A324, 301.
- 22. DOWSON, D. AND SWALES, P.D. (1967) An Elastohydrodynamic Approach to the Problem of the Reciprocating Seal. 3rd. Int. Conf. Fluid Sealing, BHRA, Cranfield, Paper F3.
- 23. BROWNE, A.L. AND HAYS, D.F. (1974) *Physics* of *Tire Traction*. New York: Plenum Press.
- FIELD, G.J. AND NAU, B.S. (1973) Lubrication of Rectangular Rubber Seals under Reciprocation Motion. BHRA Fluid Engineering Report RR1199, Cranfield, UK.
- MALAYSIAN RUBBER PRODUCERS' RESEARCH ASSOCIATION (1976) Seminar on Rubber Friction.