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An Investigation, using Finite Element Analysis, of the Effect of Swaging on Stiffnesses of Rubber Bushes

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A common practice in the manufacture of cylindrical rubber bushes is to apply a radial contraction to the outer sleeve or a radial expansion to the inner sleeve after moulding – a process known as swaging. This relieves tensile stresses which arise due to differential thermal contraction of the rubber and the sleeves and, if a sufficiently large swaging strain is applied, this compresses the bush which improves its fatigue life. A Finite Element Analysis (FEA) investigation of the effect of swaging on the small strain stiffnesses of some representative, initially stress-free, bushes was carried out. The axial and torsional stiffnesses reduce slightly as the swaging strain increases, contrary to the effect of dimensional changes alone, whereas the radial and tilting stiffnesses increase as predicted from dimensional changes. The reduction in the axial and tilting stiffness was attributed to the effect of compressive stresses introduced by swaging. An approximate estimate of the bush stiffnesses may be obtained from existing theoretical formulae if the original dimensions are used in the calculation of the axial and torsional stiffness and the swaged dimensions are used in the calculation of radial and tilting stiffness.

Key words: Finite Element Analysis (FEA); rubber; bush; mounting; stiffness; swaging; calibration; Abaqus

A rubber bush consists of a hollow rubber cylinder bonded on the inside and outside to rigid metal sleeves. Such bushes are widely used as mountings to provide compliance. Sometimes voids in the rubber, running along the length of the bush, are introduced to give a directional dependence to the radial stiffness. Typical bushes are shown in *Figure 1*.

Because the rubber is hot-bonded to both the inner and outer sleeves during moulding, tensile stresses develop as the bush cools due to greater thermal shrinkage of the rubber compared to the metal and these stresses can be sufficient to cause debonding in long bushes[1]. Therefore, a common practice in the manufacture of bushes is to reduce the diameter of the outer sleeve after moulding by forcing the bush through a slightly undersized conical die or by applying radial pressure hydraulically *via* a collet. Alternatively, the inner sleeve may be expanded by forcing a drift through the bush[2]. This process is known as swaging. The thermal stresses are relieved if the annular rubber thickness is reduced by around 6% in a long bush. However, Goldschmidt[2] recommended that the swaging strain should be between 10 and 15% of the annular thickness of

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the rubber because compressing the rubber improves the fatigue life of the bush.

METHODS

Four bush geometries were investigated. The unvoided bushes were simple plane ended hollow cylinders, assumed bonded to rigid inner and outer sleeves. The cross-section of the voided bush is shown in *Figure 2*. The ends of commercial bushes are often shaped such that the rubber is recessed around the centre of the annulus and extends further near the sleeves[2]. This is helpful in preventing the rubber from bulging beyond the ends of the sleeves, avoids a stress singularity at the bond and ensures that tensile stresses in the rubber at the ends of the bush are reduced or eliminated, which in turn inhibits crack propagation and improves the fatigue life of the bush. However, in order to keep the current investigation as clear and simple as possible and because the FEA data will be compared with analytical formulae which ignore end effects, such details were not modelled in the present study. The bush dimensions are given in *Table 1*. The commercial FEA software, Abaqus, version 6.5 was used for the analyses. The rubber was modelled throughout as a neo-Hookean material[3] with a shear modulus of 1 MPa and a bulk modulus of 2000 MPa.

The stiffnesses to be calculated are defined in *Table 2*. The geometry for the FE models depended on the symmetry which could be exploited for each stiffness measurement and is summarised in *Table 3*.

Figure 1. Rubber bushes.

TABLE 1. BUSH DIMENSIONS

Inner radius, (mm)	Outer radius, (mm)	Length, (mm)	Angle cut away at void, (degrees)
10	15	20	–
10	15	50	–
10	20	20	–
10	15	20	40

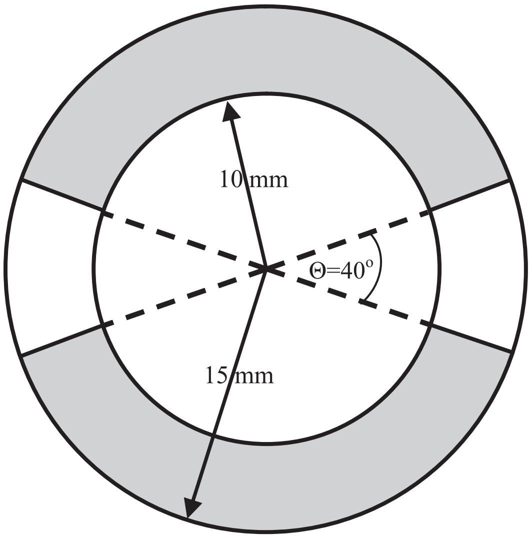


Figure 2. Cross-section geometry of voided bush.

Ten quadrilateral (2D) or brick (3D) solid elements were used through the annular thickness and at least 78 elements were used around a full circumference. Linear, reduced integration and hybrid elements were normally used[4]. Some analyses were also run with other element types; the agreement was always within 2%.

It is straightforward to model thermal contraction with Abaqus. However, in the current study only the effect of swaging on initially unstressed bushes was considered. The interpretation of these data for the case of bushes which are under thermally induced tensile stresses when the swaging is applied is considered in the Discussion section.

For the unvoided bushes, two possibilities concerning the rubber bulge at the ends of the bush, following swaging, were modelled. In the first case, ("bulging allowed"), no restrictions were placed on the bulge. In the second case, ("no bulge"), the sleeves were modelled as extending beyond the length of the bush so that bulging beyond the inner and outer radii was suppressed. This "no bulge" condition was also used for the voided bush.

For the "bulging allowed" models, boundary conditions were used to model the sleeves and impose the swaging strains and deformations required to obtain the stiffnesses. For the axisymmetric "no bulge" models, rigid bodies were introduced to represent the inner and outer sleeves. These were tied to the corresponding rubber surfaces and a contact interaction was defined to prevent the rubber penetrating the rigid bodies. For simplicity, sliding of the rubber against the sleeve was assumed to be frictionless. For the 3D "no bulge" models, a rigid body was again used to model the inner sleeve but, because the outer sleeve must reduce its radius due to swaging, it was modelled as a deformable shell and the displacements of all degrees of freedom of all nodes were defined accordingly in the boundary conditions.

TABLE 2. DEFINITIONS OF STIFFNESS

Axial	Axial displacement of one sleeve relative to the other sleeve.	F/x F = axial force, x = axial displacement
Torsional	Rotation of one sleeve about the symmetry axis with other sleeve fixed.	M/θ M = applied torque θ = angle of twist
Radial	Radial displacement of one sleeve relative to the other sleeve. For a voided bush the soft direction is for radial displacement towards voids, and the stiff direction is for radial displacement perpendicular to voids.	F/x F = radial force x = radial displacement
Tilting	Rotation of sleeve about the axis through centre of bush perpendicular to the axis of symmetry, while the other sleeve is fixed. Soft and stiff directions analogous to radial stiffness.	M/θ M = applied torque θ = angle of twist

TABLE 3. DETAILS OF MESH GEOMETRY FOR FEA

Bush	Stiffness	Mesh geometry
Unvoided	Axial	Axisymmetric
Unvoided	Torsional	Axisymmetric (element with twist), half length
Unvoided	Radial	3D, half annulus, half length
Unvoided	Tilting	3D, half annulus
Voided	Axial	3D, quarter annulus
Voided	Torsional	3D, half annulus, half length
Voided	Radial	3D, half annulus*, half length
Voided	Tilting	3D, half annulus*

* A different half is required for the stiff and soft directions; in the soft direction the symmetry plane passes through the voids, in the stiff direction it is perpendicular to this.

The swaging strains were applied by reducing the radius of the outer sleeve while the inner sleeve position was fixed. The small-strain stiffnesses were calculated from the reaction force or moment arising from a small displacement or rotation of the inner sleeve whilst the position of the outer sleeve was fixed. For the axial and radial stiffness measurements, displacements of 0.05mm were applied, while for the torsional and tilting stiffnesses, rotations of 0.005 radians were applied. Since only small-strain stiffnesses were considered, it was deemed sufficient to assume linearity and apply the deformation

TABLE 4. FORMULAE FOR BUSH STIFFNESSES

Bush	Stiffness	Formula for stiffness		Source
Unvoided	Axial	$\dfrac{2\pi GL}{\ln(R/r)}$	… 2	Rivlin[7] Adkins and Gent[1] EDNR
Unvoided	Torsional	$\dfrac{4\pi GL}{r^{-2} - R^{-2}}$	… 3	Rivlin[7] Adkins and Gent[1] EDNR
Unvoided	Radial	$GL\left[\beta_S + \left(0.169 - \dfrac{0.0356R}{r}\right)(\beta_L - \beta_S)\left(\dfrac{L}{r} - 0.5\right)\right]$	… 4	Busfield and Davies[5]
Unvoided	Tilting	see reference	… 5	Horton *et al.*[9]
Voided	Axial	$\dfrac{2GL\,(\pi - \Theta)}{\ln(R/r)}$	… 6	
Voided	Axial	$\dfrac{4GL\,(\pi - \Theta)}{r^{-2} - R^{-2}}$	… 7	
Voided	Radial, soft	$\dfrac{2GL\bar{r}}{t}\left[(2+S^2)(\pi - \Theta) - \sin\Theta\,(1 + S^2)\right]$	… 8	
Voided	Radial, stiff	$\dfrac{2GL\bar{r}}{t}\left[(2+3S^2)(\pi - \Theta) - \sin\Theta\,(1 + 3S^2)\right]$	… 9	

G = shear modulus of rubber,
L = length, R = outer radius, r = inner radius, Θ = angle subtended by one void at centre (see *Figure 2*)

$$\beta_S = \frac{80\,\pi\,(R^2 + r^2)}{25\,(R^2 + r^2)\ln(R/r) - 9\,(R^2 - r^2)} \qquad \text{…10}$$

$$\beta_L = \frac{4\,\pi\,(R^2 + r^2)}{(R^2 + r^2)\ln(R/r) - (R^2 - r^2)} \qquad \text{…11}$$

$$S = \frac{Lr\,(\pi - \Theta)}{2t\,[L + r\,(\pi - \Theta)]} \qquad \text{…12}$$

$$\bar{r} = \frac{1}{2}(R + r) \qquad \text{…13}$$

$$t = R - r \qquad \text{…14}$$

in a single increment. Likewise, a single increment was normally used for each addition of swaging strain.

RESULTS

The bush stiffnesses obtained from the FEA are plotted in *Figures 3* to *6* as a function of swaging strain expressed as a percentage of the annular thickness of rubber.

Comparison with Analytical Formulae for Unswaged Bushes

Various analytical formulae for calculating the small-strain stiffnesses of bushes have been published. Some of these formulae were used to estimate the stiffnesses of the swaged bushes analysed above. In the case of radial stiffness, the formula used[5] is based on empirical fits to FEA measurements. An analytical formula was derived theoretically by

Horton *et al.*[6] which gives very similar values. Semi-empirical formulae for the stiffnesses of voided bushes, of the type shown in *Figure 2*, were derived.

In using these formulae to calculate the stiffness of the swaged bushes, the assumption was made that the stiffness depends only on the geometry of the swaged bush and not on the residual stresses. For each swaging strain modelled, the outer diameter of the bush was recalculated and a new bush length determined based on the assumption that the rubber was incompressible. Hence;

$$L_S = \frac{L_0\,(R_0^2 - r_0^2)}{(R_S^2 - r_0^2)} \qquad \ldots 1$$

where L is the length, r and R the inner and outer radii respectively and the subscripts 0 and S represent the unswaged and swaged dimensions respectively. The stiffness formulae used in this work are summarised in *Table 4*. The stiffnesses of the bushes obtained from these formulae are also plotted on *Figures 3* to *6*.

Comparison with Theory for Swaged Bushes

Rivlin[7] considered the case of extension, inflation, torsional and axial shear of a rubber annulus. The extension and inflation serve to change the initial dimensions which, for the case of no change in the inner radius, may be likened to swaging of a lubricated, rather than bonded, bush. Rivlin's method further assumes that the bush is infinitely long or that appropriate stresses are applied to the plane ends of the bush so that they remain planar. From Rivlin's results the axial and torsional stiffness of unvoided bushes may be calculated. Hill[10] similarly derived expressions for the radial and tilting stiffness of infinitely long swaged bushes. Their expressions are given below for the case of a neo-Hookean material.

Axial stiffness = $\dfrac{2\pi G L_0}{\ln(R_0 / r_0)}$ $\qquad \ldots 15$

Equation 15 agrees with *Equation 2* for unswaged bushes.

Torsional stiffness = $\dfrac{2\pi G L_0 r_0^2\,(R_0^2 - R_S^2)}{(R_0^2 - r_0^2)\,\ln(R_0 / R_S)}$ $\qquad \ldots 16$

As required, as $R_0 - R_S \to 0$ *Equation 16* reduces to *Equation 3*.

Radial stiffness = $\dfrac{16\pi G L_0 K^2\,[(R_0^2 + K)^2 - (r_0^2 + K)^2]}{4(R_0^4 - r_0^4)\,[(R_0^2 + K)^2 - (r_0^2 + K)^2] - [\varphi(R_0) - \varphi(r_0)]^2}$ $\qquad \ldots 17$

where, $K = \dfrac{r_0^2\,(R_0^2 - R_S^2)}{(R_S^2 - r_0^2)}$ $\qquad \ldots 18$

and, $\varphi(z) = K^2 \ln(z + \sqrt{z^2 + K}) - z\sqrt{z^2 + K}(2z^2 + K)$ $\qquad \ldots 19$

As $R_S - R_0 \to 0$ *Equation 17* reduces to the usual equation for the radial stiffness of infinitely long bushes[1,8].

Tilting stiffness = $\dfrac{k_r L_0^2\,(R_0^2 - r_0^2)^2}{12(R_S^2 - r_0^2)^2}$ $\qquad \ldots 20$

where, k_r is the radial stiffness.

These expressions were evaluated and are also plotted in *Figures 3* to *6*.

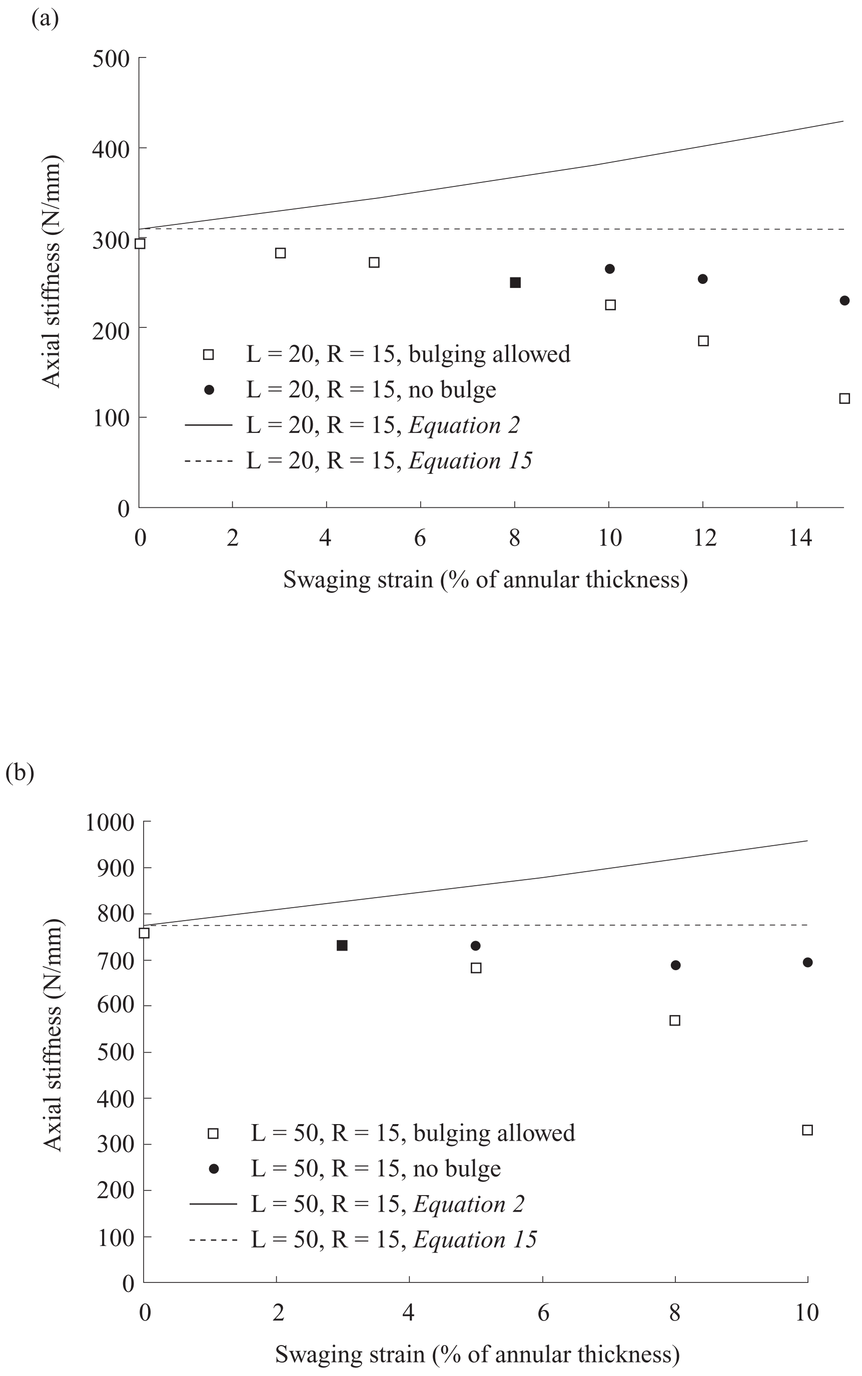
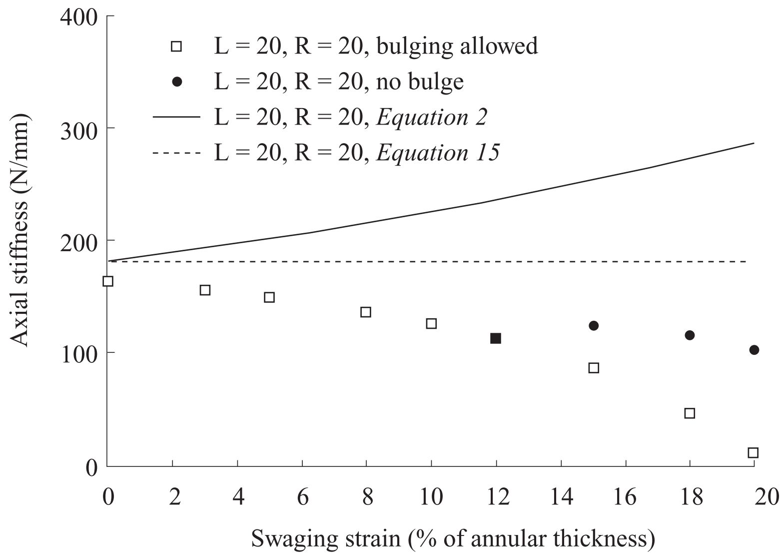


Figure 3. Axial stiffness of swaged bushes. L is length and R is outer radius of bush (in mm for shear modulus of 1 MPa and inner radius of 10 mm).

(c)



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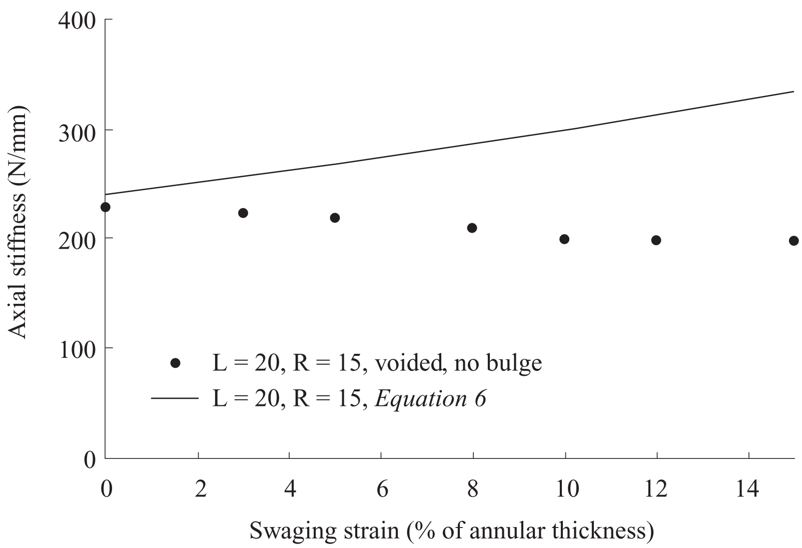
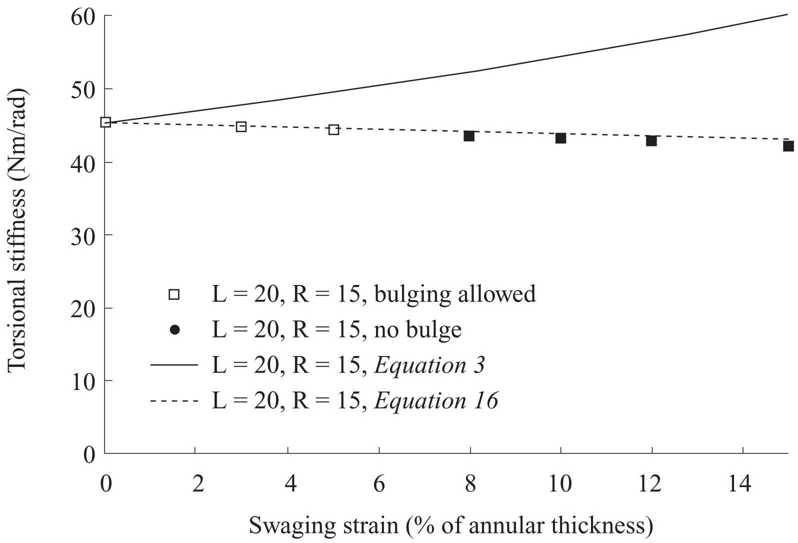


Figure 3. Axial stiffness of swaged bushes. L is length and R is outer radius of bush (in mm for shear modulus of 1 MPa and inner radius of 10 mm).

(a)



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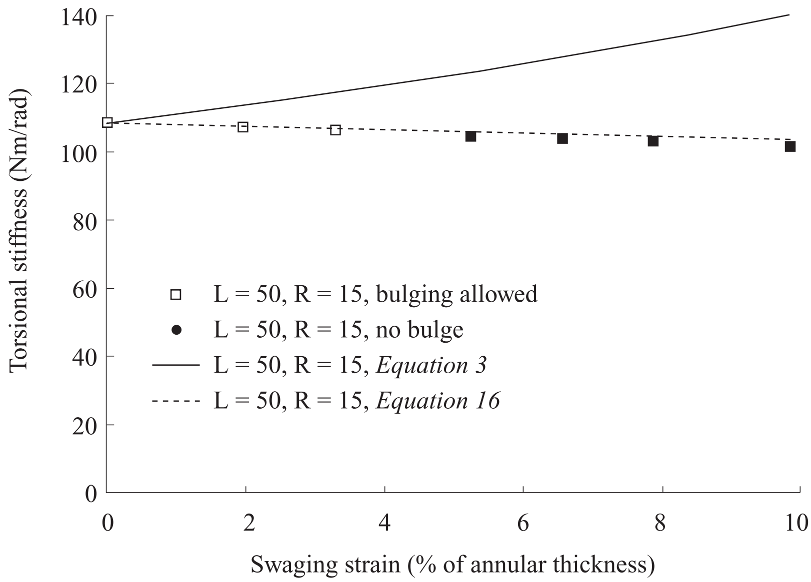
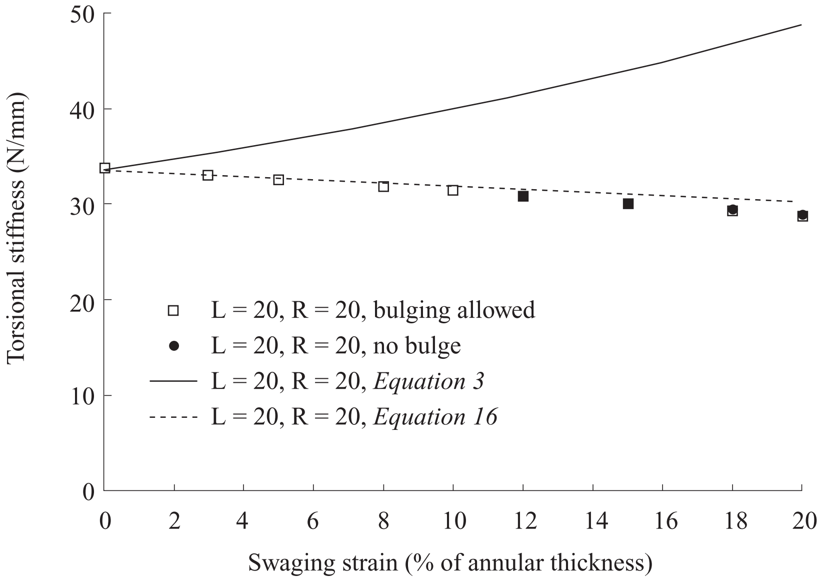


Figure 4. Torsional stiffness of swaged bushes. L is length and R is outer radius of bush (in mm for shear modulus of 1 MPa and inner radius of 10 mm).

(c)



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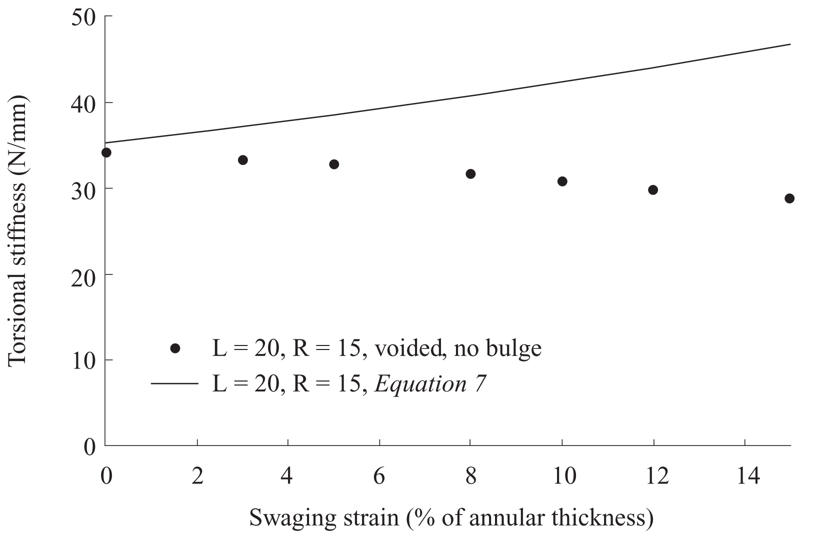
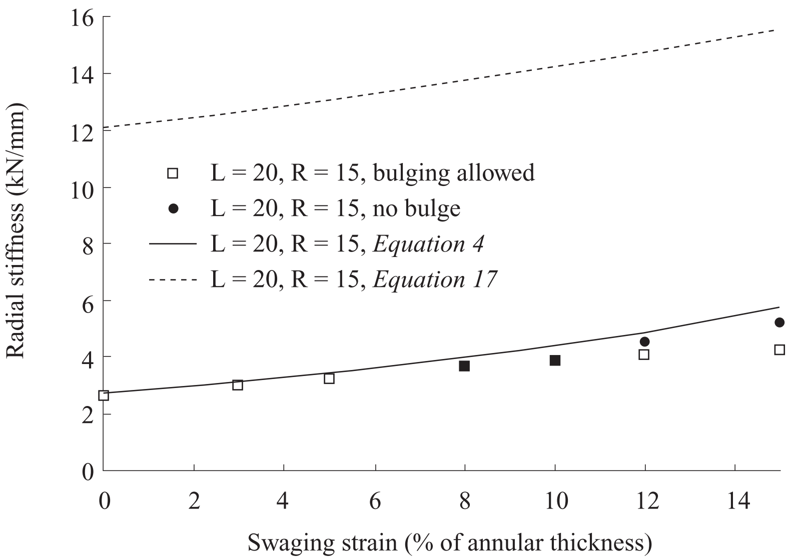


Figure 4. Torsional stiffness of swaged bushes. L is length and R is outer radius of bush (in mm for shear modulus of 1 MPa and inner radius of 10 mm).

(a)



(b)

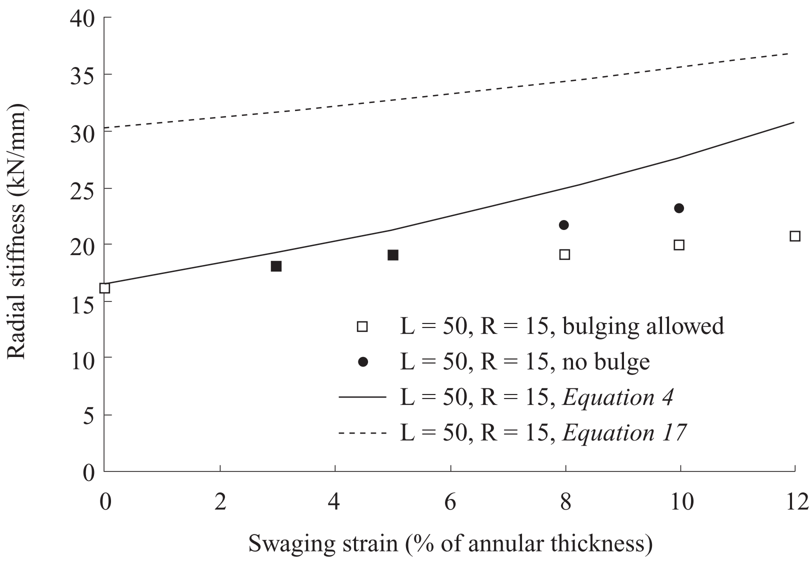
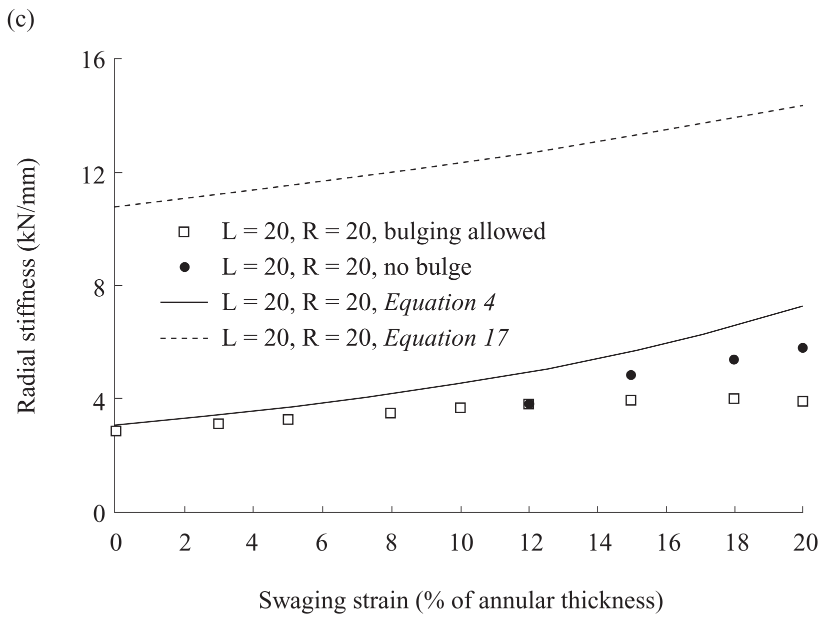


Figure 5. Radial stiffness of swaged bushes. L is length and R is outer radius of bush (in mm for shear modulus of 1 MPa and inner radius of 10 mm).



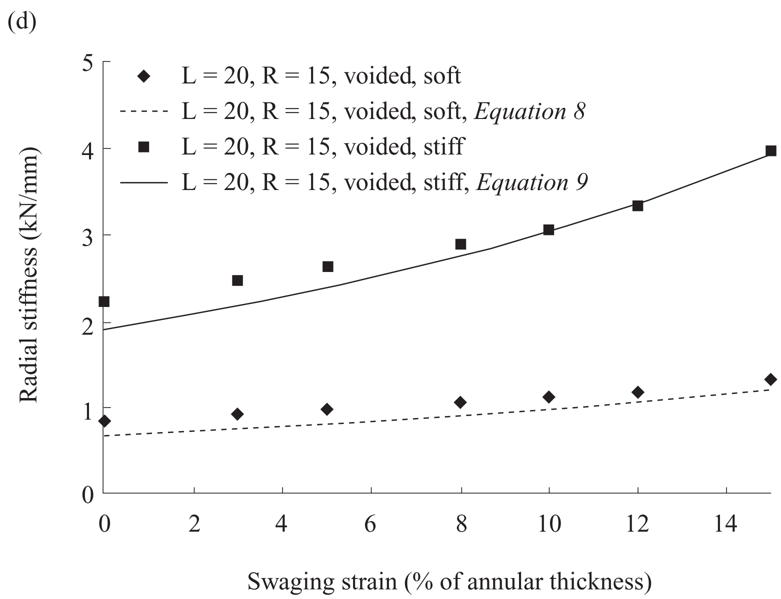
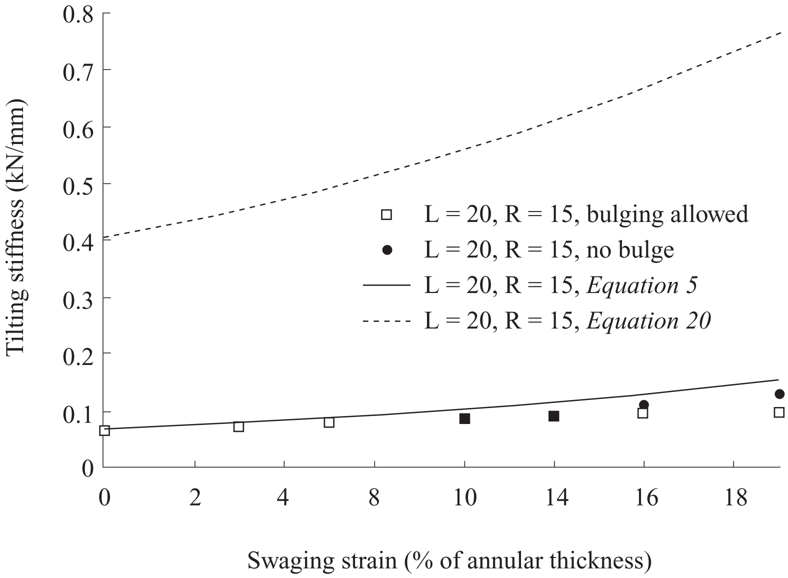


Figure 5. Radial stiffness of swaged bushes. L is length and R is outer radius of bush (in mm for shear modulus of 1 MPa and inner radius of 10 mm).

(a)



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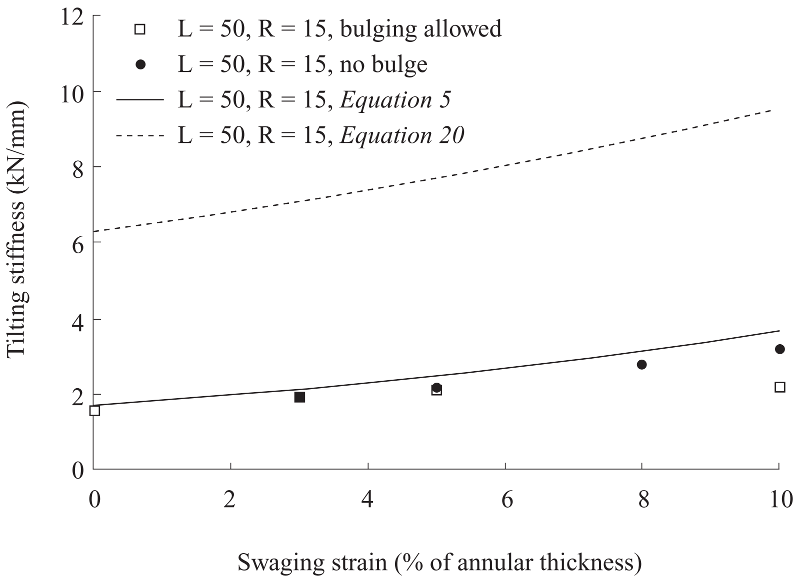
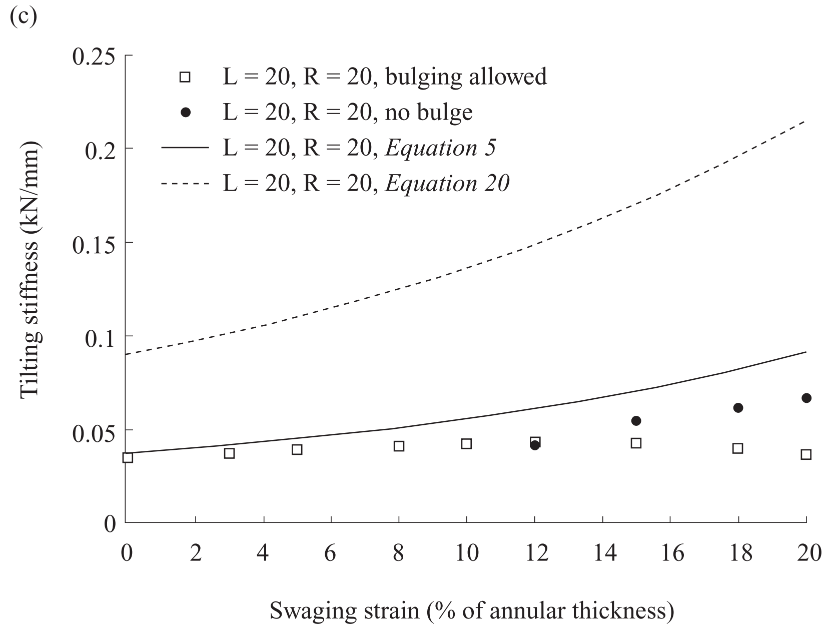


Figure 6. Tilting stiffness of swaged bushes. L is length and R is outer radius of bush (in mm for shear modulus of 1 MPa and inner radius of 10 mm).



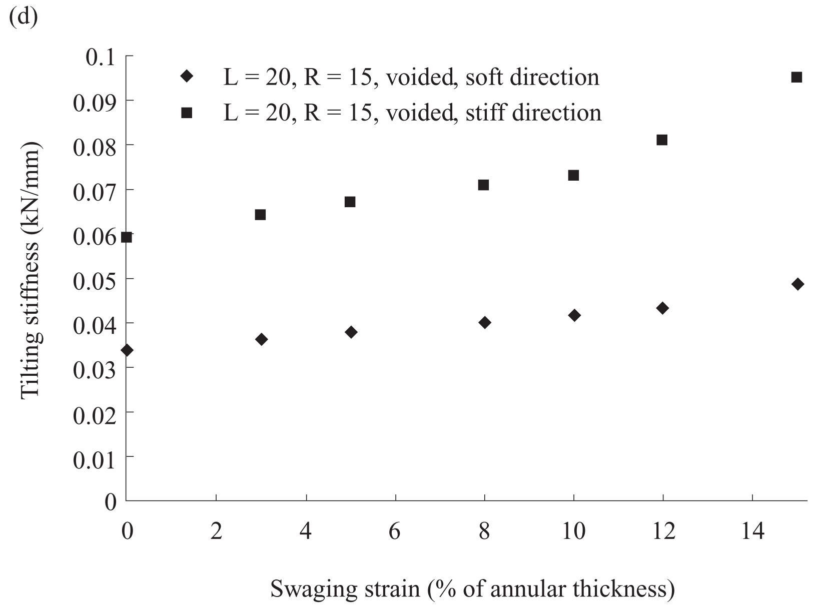


Figure 6. Tilting stiffness of swaged bushes. L is length and R is outer radius of bush (in mm for shear modulus of 1 MPa and inner radius of 10 mm).

DISCUSSION

The axial and torsional stiffness fall as the swaging strain increases, but the simple formula for unswaged bushes, ignoring the effect of stress but allowing for dimensional changes, suggests a rising stiffness. Rivlin's[7] theory predicts that the axial stiffness is given by the simple formula (*Equation 2*) using the original dimensions. The FEA results indicate that the stiffness is less than this, presumably mainly due to the free ends of the bush which the theory does not address, in the case where bulging is not constrained, the FEA results reveal a large discrepancy. Rivlin's equation, (*Equation 16*), gives an accurate prediction of the torsional stiffness of unvoided bushes. The fall in axial and torsional stiffness may be similar to the effect a precompression has on reducing the shear stiffness of bonded blocks[11]. It is not really possible to make a quantitative comparison with the work of Fan *et al.*[11] because, except for the bush with voids, the bushes represent higher shape-factor blocks than they investigated and the swaging strains were smaller than the compressions they used, but the results are not inconsistent. The axial softening is more pronounced at smaller swaging strains for the higher shape-factor bushes. For both axial and torsional stiffness, it is suggested that the compressive stresses arising from the swaging are partially relieved by the axial or torsional deformation and this is the reason for the lower stiffness.

The radial and tilting stiffnesses increase as the swaging strains increase, as would be expected from the dimensional changes, but the results suggest that use of formulae based on the swaged dimensions tends to overestimate the stiffness slightly, at least for the unvoided bushes. This is more likely to be due to these approximate formulae providing an upper-bound or overestimate of the stiffness than because the swaging stresses cause any softening. The rise in stiffness with swaging is also predicted by Hill's equations[10] but the assumption of an infinitely long bush is too unrealistic for these equations to provide a useful estimate of stiffness.

The present analyses have ignored the effect of cooling after moulding; this introduces residual stresses which would tend to offset those introduced by swaging. For an infinitely long bush, there exists a swaging strain which exactly compensates for the thermally induced stresses. For shorter bushes, there will be end effects so that the thermal stresses cannot be exactly eliminated by a (radial) swaging strain.

The thermal stresses will be smaller in very short or voided bushes because they can be relieved by thermal contraction at the ends of the annulus or at the voids. However, to a first approximation the swaging strain applied in this work may be interpreted as the strain in excess of that required to compensate for thermal shrinkage. Alternatively, the thermal stress could be interpreted as a negative swaging of a slightly thinner bush. Consistent with this, Ollson *et al.*[12] have found that thermal stresses due to cooling reduced the radial stiffness of an unswaged bush.

It is proposed that a reasonable estimate of the stiffnesses of swaged bushes may be made from appropriate formulae for unswaged bushes if the stress-free dimensions are used in the calculation of axial and torsional stiffness and the swaged dimensions are used in the calculation of radial and tilting stiffness. This rule is applicable for bushes with and without voids. A better prediction of the torsional stiffness may be made using Rivlin's equation (*Equation 16*)[7].

The FEA was carried out using a neo-Hookean rubber model; this was also assumed in applying the formulae. It is generally believed that the influence of the second strain invariant (I_2) on the stress-strain behaviour of rubber is small and therefore best ignored, especially when the objective is to provide simple design formulae that are in any case only approximate[13,14]. However, some of the derivations have been carried out for a general strain energy function or at least for the special case of a Mooney model. An interesting observation arising from Hill's paper[10] is that, for swaging strains greater than 35% and a Mooney model, a steep increase in torsional stiffness is expected. This effect is not observed with the neo-Hookean model. However, since the effect is not evident at lower swaging strains it is unlikely to be of practical importance.

CONCLUSIONS

FEA has demonstrated that the axial and torsional stiffness of swaged bushes reduces as the swaging strain increases. The reduction is particularly pronounced in the axial stiffness of unvoided bushes. It is attributed to an effect of compressive residual stresses introduced by swaging. FEA has also demonstrated that the radial and tilting stiffness of swaged bushes increases with swaging strains as would be expected from the dimensional changes. The effect of residual stresses on these stiffnesses is small.

A reasonable estimate of the stiffness of swaged bushes may be made from analytical formulae if the unstressed dimensions are used in the calculation of the axial and torsional stiffness and the swaged dimensions are used in the calculation of radial and tilting stiffness. An equation of Rivlin[7] may be used to give a better prediction of the torsional stiffness of swaged bushes.

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