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Use of Approximate Calculations and Finite Element Analysis to Estimate the Stiffness of Rubber Bushes and Cylindrical Mountings

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Rubber bushes consist of a hollow rubber cylinder bonded on the inside and outside to rigid metal sleeves and are widely used as mountings to provide compliance. Four modes of deformation are commonly defined: axial, torsional, radial and tilting. Formulae, available in the literature for calculating the small strain stiffnesses of such bushes, are briefly reviewed. Sometimes two voids are introduced on opposite sides, running along the length of the bush, to give a directional dependence to the radial and tilting stiffness. An approximate theoretical treatment for such bushes is presented, together with predictions of the stiffnesses from FEA. Mountings in which the inner and outer sleeves are connected by two blocks of rubber, in effect bushes with very large voids, are also considered. It is shown that reasonable predictions of the bush stiffnesses may be obtained from the approximate formulae. Such formulae may therefore be used in design calculations to provide an initial estimate of the required dimensions of the bush and FEA may then be used to refine the initial design.

Keywords: Finite Element Analysis (FEA); bush; mounting; stiffness; rubber

A rubber bush consists of a hollow rubber cylinder bonded on the inside and outside to rigid metal sleeves. Such bushes are widely used as mountings to provide compliance. Sometimes two voids are introduced on opposite sides, running along the length of the bush, to give a directional dependence to the radial stiffness. Typical bushes are shown in *Figure 1a*. Another widely used mounting consists of cylindrical steel sleeves connected by two blocks of rubber as shown in *Figure 1b*. Snubbers are often incorporated to control the large strain radial stiffness in various directions.

The bush or mounting may be deformed in various ways and it is convenient to define four modes of deformation as given in *Table 1*. In service the bush may be subjected to a combination of these.

In this paper, existing formulae for the stiffnesses of solid bushes are reviewed and some approximate formulae for the stiffnesses of voided bushes and shear block mountings are derived. FEA is used to verify the formulae and some empirical modifications are introduced to achieve better agreement with the FEA results. In all cases only the

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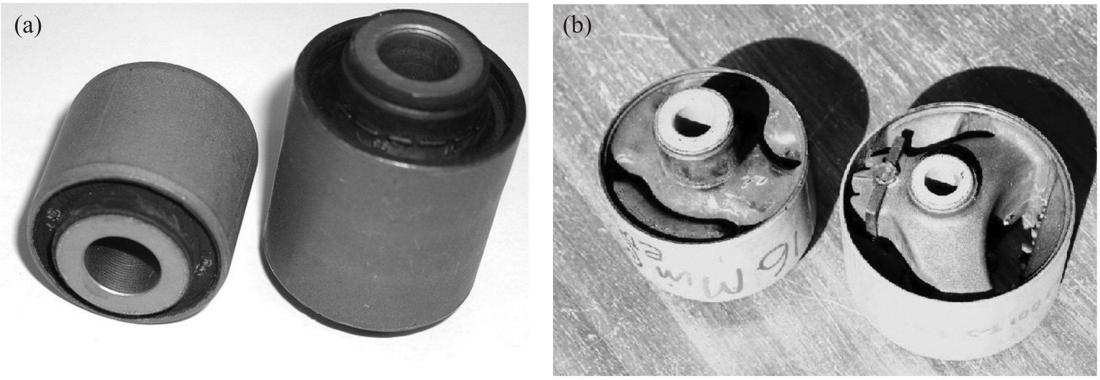

Figure 1. Rubber bushes (a) conventional voided bushes; (b) cylindrical shear mountings

TABLE 1. DEFINITIONS OF STIFFNESS

Axial	Axial displacement of one sleeve relative to the other sleeve	$k_a = F_a/x_a$ F_a = axial force, x_a = axial displacement
Torsional	Relative rotation of one sleeve to the other about the symmetry axis.	$k_{tor} = M_{tor}/\theta_{tor}$ M_{tor} = applied torque θ_{tor} = angle of twist
Radial	Radial displacement of one sleeve relative to the other sleeve. For a voided bush, the soft direction is for radial displacement towards the voids, and the stiff direction is for radial displacement perpendicular to the voids.	$k_r = F_r/x_r$ F_r = radial force x_r = radial displacement
Tilting	Relative rotation of one sleeve to the other about an axis through the centre of the bush perpendicular to axis of symmetry. The soft and stiff directions are analogous to radial stiffness.	$k_{tilt} = M_{tilt}/\theta_{tilt}$ M_{tilt} = applied torque θ_{tilt} = angle of twist

initial small-strain stiffnesses are considered and the rubber is treated throughout as a perfectly elastic neo-Hookean material so that the effects of kinematic and material non-linearity, dynamic, time-dependent and hysteretic effects are all beyond the scope of this paper.

SIMPLE ANNULAR BUSHES

The simplest type of bush is a rubber annular prism, bonded on its inner and outer curved surfaces to stiff metal sleeves. This case has been considered analytically by various workers and formulae derived for the stiffnesses.

Rivlin[1] derived an expression for the axial stiffness of a bush, assuming a uniform state of simple shear along the length of the bush. For a bush of internal and external radii, r_i and r_o respectively, length, L, and shear modulus, G, his expression is

$$k_a = \frac{2\pi G L}{\ell n(r_o / r_i)} \quad \ldots 1$$

which is appropriate for long bushes. For short bushes, the contribution due to bending should also be taken into account. Modifications of *Equation 1* to allow for bending have been proposed by Adkins and Gent[2] and Busfield and Davies[3].

Rivlin[1] also derived an expression for the torsional stiffness which is given by:

$$k_{tor} = \frac{4\pi G L}{r_i^{-2} - r_o^{-2}} \quad \ldots 2$$

for a small angle of rotation.

Expressions for the radial stiffness of very long and very short bushes were derived by Stevenson[4]. An interpolation for bushes of finite length was developed empirically by Busfield and Davies[3] from FEA simulations. The problem for bushes of finite length was solved theoretically by Hill[5] but his solution involves infinite series of Bessel functions and is not easy to evaluate exactly. An approximate theoretical treatment is given by Horton *et al.*[6]. Evaluation of the expressions of Busfield and Davies and Horton *et al.* for bushes of various geometries indicates that they give similar results, which are also in good agreement with the FEA results of Busfield and Davies and thus either expression may be used to give an adequate engineering value for the radial stiffness of solid bushes.

Horton *et al*. have also derived an expression for the tilting stiffness of bushes[7]. Comparison with FEA has indicated that this expression, although not exact, is adequate for engineering calculations.

VOIDED BUSHES

To simplify the mathematics the geometry shown in *Figure 2* was assumed. The length of the bush perpendicular to the plane of the figure is L.

Axial Stiffness

Consider an annulus of thickness δ_r at a radial distance r from the centre of the bush. An axial force F_a produces a deflection δx_a across the element of height δ_r. We assume the bush is long enough to be in simple shear and ignore edge effects due to the free surfaces on the inside of the voids. Then the shear strain, γ_a (r), is given by:

$$\gamma_a(r) = \frac{d x_a}{d r} = \frac{\tau_a(r)}{G} = \frac{F_a}{2(\pi - \Theta) r L G} \quad \ldots 3$$

where $\tau_a(r)$ is the axial shear stress in the annulus Hence the axial stiffness is given by integrating *Equation 3* between r_i and r_o:

$$k_a = \frac{F_a}{x_a} = \frac{2(\pi - \Theta) G L}{\ell n(r_o / r_i)} \quad \ldots 4$$

Torsional Stiffness

The basic calculation for solid bushes is given in Göbel[8]. Consider an annulus at a distance r from the axis of the bush. Again, we ignore edge effects due to the free surfaces on the inside of the voids. The shear stress-strain

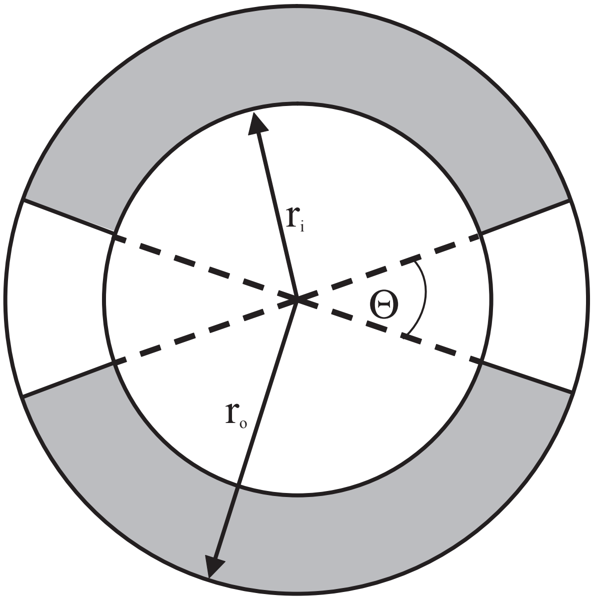

Figure 2. Simplified cross-sectional geometry of voided bush.

relationship for rotation of the outer sleeve relative to the inner sleeve is

$$\tau_{tor}(r) = G\,\gamma_{tor}(r) \qquad \ldots 5$$

Hence

$$\frac{M_{tor}}{2L(\pi-\Theta)r^2} = G\,r\frac{d\theta_{tor}}{dr} \qquad \ldots 6$$

where M_{tor} is the torque (independent of r) and θ_{tor} is the (small) angle of twist. The torsional stiffness, k_{tor}, is obtained by integrating between r_i and r_o so that

$$k_{tor} = \frac{M_{tor}}{\theta_{tor}} = \frac{4GL(\pi-\Theta)}{r_i^{-2} - r_o^{-2}} \qquad \ldots 7$$

Radial Stiffness

This problem is more difficult to solve theoretically so a very approximate approach was used, based on that of Göbel[8] for bushes without voids.

Consider the shaded element shown in *Figure 3*. The outside edge is displaced a radial distance x_r under a force F_r. The compressive and shear forces acting on the element are given respectively as

$$\delta F_c = \sigma_c\,Lr\,\delta\alpha \qquad \ldots 8a$$

$$\delta F_s = \sigma_s\,Lr\,\delta\alpha \qquad \ldots 8b$$

where σ_c and σ_s are the compressive and shear stresses respectively in the element, and

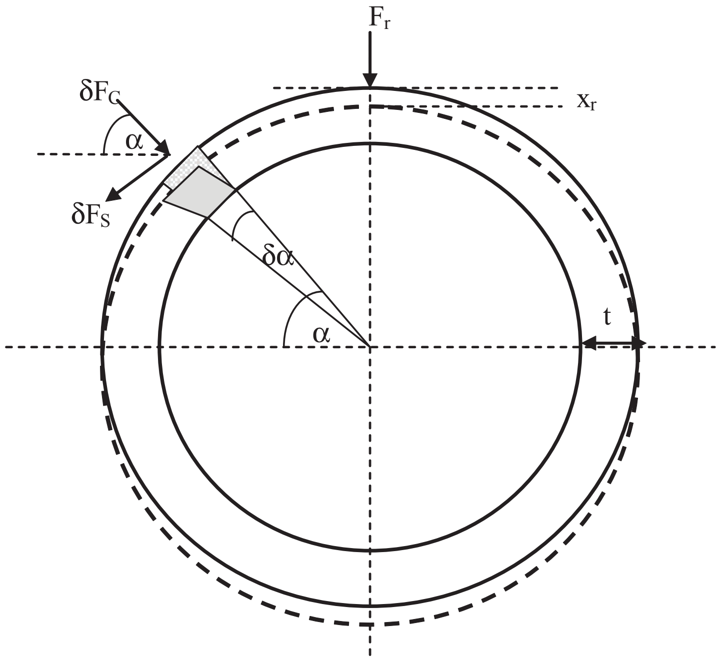

Figure 3. Diagram of bush under radial deflection.

r is the bush radius. We assume that the stresses are constant over the area of the element; hence we assume that the thickness of the rubber annulus is small.

We may write

$$\sigma_c = E_c \ \varepsilon_c = \frac{E_c x_r \sin\alpha}{t} \qquad \text{… 9a}$$

$$\sigma_s = G \ \varepsilon_s = \frac{G x_r \cos\alpha}{t} \qquad \text{… 9b}$$

where E_c is the compression modulus and t is the thickness of the rubber annulus. Hence, the total vertical force acting on the element is given by

$$\delta F_r = \delta F_c \ \sin\alpha + \delta F_s \ \cos\alpha = \frac{L r x_r}{t} \left(E_c \ \sin^2\alpha + G\cos^2\alpha\right)\delta\alpha \qquad \text{… 10}$$

For a bush with voids, *Equation 10* may be integrated between appropriate limits to give approximate expressions for the radial stiffness in "soft" and "stiff" directions. In the soft direction

$$F_{r\,soft} = 2\int_{\alpha=\frac{-\pi}{2}+\frac{\Theta}{2}}^{\alpha=\frac{\pi}{2}-\frac{\Theta}{2}} dF_r = \frac{x_r\,Lr}{t}\left[(E_c+G)\,(\pi-\Theta)-(E_c-G)\sin\Theta\right] \qquad \dots 11$$

and in the stiff direction

$$F_{r\,stiff} = \int_{\alpha=\frac{\Theta}{2}}^{\alpha=\pi-\frac{\Theta}{2}} dF_r + \int_{\alpha=\pi+\frac{\Theta}{2}}^{\alpha=2\pi-\frac{\Theta}{2}} dF_r = \frac{x_r\,Lr}{t}\left[(E_c+G)\,(\pi-\Theta)+(E_c-G)\sin\Theta\right] \qquad \dots 12$$

For the case of bushes with voids, the rubber behaves more like a rectangular block than an infinite strip so E_c is approximately given by[9]

$$E_c = 3G\,(1+2S^2) \qquad \dots 13$$

where the shape factor, S, is given by

$$S = \frac{Lr\,(\pi-\Theta)}{2t\,\left(L+r\,(\pi-\Theta)\right)} \qquad \dots 14$$

The reduced stiffnesses, defined as $k_r^* = k_r/(GL) = F_r/(GLx_r)$, are then

$$k_r^{*}{}_{soft} = \frac{2r}{t}\left[(2+3S^2)\,(\pi-\Theta)-(1+3S^2)\sin\Theta\right] \qquad \dots 15$$

$$k_r^{*}{}_{stiff} = \frac{2r}{t}\left[(2+3S^2)\,(\pi-\Theta)+(1+3S^2)\sin\Theta\right] \qquad \dots 16$$

where r is the mean radius of the rubber annulus, given by

$$r \approx (r_i + r_o)/2 \qquad \dots 17$$

and t is the annular thickness of the rubber, given by

$$t = r_o - r_i \qquad \dots 18$$

FEA

The usefulness of the calculation for the radial stiffness was assessed by comparing to FEA simulations. The FEA was carried out using the commercial software, MSC.Marc. Bushes of finite length were considered so 3D FEA models were required. In a radially deformed bush there are two symmetry planes and these were exploited so that one quarter of the bush was modelled. Eight-noded, solid Herrmann elements were used with full integration. At least five elements were used through the thickness of the rubber (in the radial direction) and the number of elements around the circumference and along the length was chosen to give reasonably cubic elements. The rubber was modelled as a neo-Hookean material with a shear modulus of 1 MPa and a bulk modulus of 2000 MPa. In one case, where the rubber was most constrained ($L = 100$ mm, $r_o = 15$ mm), a rubber with a bulk modulus ten times higher was also modelled, but gave almost identical results. Hence the effect of bulk compression on the initial stiffness of voided bushes is generally insignificant. A vertical displacement of 1% of the rubber annular thickness was applied to all nodes around the outside of the bush, whilst the nodes on the inside were held stationary. The reduced stiffnesses were calculated from the resulting reaction force and the bush dimensions.

RESULTS

Equations 15 and *16* are compared to the FEA results in *Table 2*. In all cases the angle subtended at the centre by one void, Θ, was 40º and the inner radius of the bush, r_i was 10 mm.

DISCUSSION

Bearing in mind that the theory is rather crude, the agreement with the FEA is reasonable. The main failing is a too steeply rising reduced

TABLE 2. RADIAL STIFFNESS OF VOIDED BUSHES

Dimensions of bush $r_i = 10$ mm, $\Theta = 40º$		Reduced radial stiffness				
		Stiff direction		Soft direction		
Length L mm	Outer radius r_o mm	Equation 16	FEA	Equation 15	Equation 20	FEA
10	15	54	61	37	26	33
20	15	95	115	61	34	44
50	15	194	236	118	54	55
100	15	281	297	169	71	59
10	20	21	23	15.2	13.6	14.7
20	20	28	31	19.5	15.0	16.7
50	20	48	54	31	18.8	20
100	20	67	70	42	22.4	21
10	40	9.5	8.8	7.3	7.1	6.1
20	40	10.2	9.2	7.6	7.3	6.2
50	40	12.5	11.0	9.0	7.7	6.8
100	40	15.4	13.6	10.7	8.3	7.4

stiffness with increasing bush length predicted by the theory for the soft direction.

The theory over-predicts the constraint due to the bond in the soft direction because the compression is greatest near the void, where the rubber is free to bulge, and zero half way between the voids where the rubber is most constrained. To correct for this we could introduce a dependence of the shape factor on α but instead it was found empirically that a reasonable fit was obtained if *Equation 13* was replaced with

$$E_c = 3G\left(1 + \frac{2}{3}S^2\right) \qquad \dots 19$$

which presumably represents an appropriately weighted average value, so that *Equation 15* becomes

$$k_r{}^*{}_{soft} = \frac{2r}{t}\Big[(2+S^2)(\pi-\Theta) - \sin\Theta(1+S^2)\Big] \qquad \dots 20$$

This Equation is also experessed in *Table 2*.

If *Equation 20* is used in place of *Equation 15* for the soft direction, the theory is able to predict the bush radial stiffness to within about 20% in all cases investigated.

FEA simulations of axial, torsional and radial deformations of a voided bush with an inner radius of 10mm, outer radius of 15mm, length of 20mm and angle cut out Θ (*Figure 2*) of 40º have been reported elsewhere[10]. They showed good agreement with *Equations 4, 7, 16* and *20*.

SHEAR BLOCK MOUNTINGS

We consider a mounting with two identical rubber blocks connecting the inner and outer sleeves so that the cross section is as shown in *Figure 4*. The length of the mounting, perpendicular to the Figure, is L.

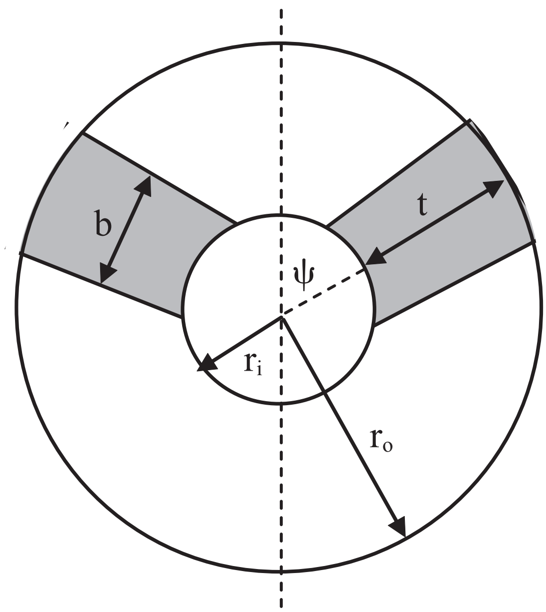

Figure 4. Cross-section of a shear block mounting.

Axial Stiffness

The simplest approach is to treat the component as two shear blocks. Then, the stiffness, including the contribution due to bending, is given approximately by Rivlin and Saunders[11].

$$k_a = \frac{2GLb}{t[1+(t^2/36\kappa^2)]} \qquad \dots 21$$

where b is the width of the block, t is the thickness of the block and κ is the radius of gyration of the cross-section about the neutral axis. We shall introduce $t = r_o - r_i$ and $\kappa^2 = L^2/12$ so that *Equation 21* becomes

$$k_a = \frac{6GL^3b}{(r_o - r_i)[3L^2 + (r_o - r_i)^2]} \qquad \dots 22$$

Torsional Stiffness

We use a similar approach to that for the voided bush, but in this case we take the area of one shear block as bL so that *Equation 6* becomes

$$\frac{M_{tor}}{bLr} = Gr\frac{d\theta_{tor}}{dr} \qquad \dots 23$$

for one block. Hence, the torsional stiffness of the two-block mount, excluding the effect of bending or warping[12], is obtained by integration of *Equation 23*:

$$k_{tor\,s} = \frac{2M_{tor}}{\theta_{tor}} = \frac{2bLG}{r_i^{-1} - r_o^{-1}} \qquad \dots 24$$

In this type of mounting the thickness t of the block is likely to be at least as big as the breadth b (*Figure 1b*). Therefore, we might expect *Equation 24* to overestimate the torsional stiffness because it assumes a state in simple shear and ignores bending. To estimate a bending correction we treat each block as a beam under a concentrated shear load F_{tor}. For a beam fixed at both ends, ignoring details of the boundary conditions, the standard result from beam theory gives the deflection due to bending, x_b, as[11]

$$x_b = \frac{F_{tor}t^3}{3LGb^3} \qquad \dots 25$$

Equation 25 is for a single block. We introduce $F_{tor} \approx M_{tor} / r$, $x_b \approx r\theta_b$, , where θ_b is the angle of rotation contributed by the bending compliance and $t = r_o - r_i$ so that

$$k_{tor\,b} = \frac{2M_{tor}}{\theta_b} = \frac{6LGb^3r^2}{(r_o - r_i)^3} \qquad \dots 26$$

It is not clear what is the appropriate radius to use for r, nevertheless it would be reasonable to assume that it may be approximated by the mean radius, $(r_o + r_i) / 2$. We also assume, following Rivlin and Saunders[11], that the torsional stiffness is given by summing the compliances due to shear and bending. Hence

$$k_{tor} = \left(\frac{1}{k_{tor\,s}} + \frac{1}{k_{tor\,b}}\right)^{-1} = GLb\left[\frac{r_i^{-1} - r_o^{-1}}{2} + \frac{2(r_o - r_i)^3}{3b^2(r_o + r_i)^2}\right]^{-1} \qquad \dots 27$$

Radial Stiffness

The stiffness depends on the radial direction. The simplest approach is similar to that for inclined shear mountings[13]. We treat the blocks as though they were bonded to flat parallel plates rather than cylindrical sleeves and ignore bending. If we consider the stiffness at an angle φ to the symmetry plane (*Figure 5*) we may write

$$\begin{aligned} N_1 &= k_c x_r \cos(\psi - \varphi) \\ N_2 &= k_c x_r \cos(\psi + \varphi) \\ F_1 &= k_s x_r \sin(\psi - \varphi) \\ F_2 &= k_s x_r \sin(\psi + \varphi) \end{aligned} \qquad \ldots 28$$

where N and F are the compression and shear components respectively of the force on blocks 1 and 2 under a displacement δ in the direction of the stiffness measurement. ψ is the angle between the symmetry plane and the compression axis of the block. k_c and k_s are the compression and shear stiffnesses respectively, given by

$$k_c = \frac{3GLb}{t}\left(1 + 2S^2\right) \qquad \ldots 29$$

$$k_s = \frac{GLb}{t} \qquad \ldots 30$$

where the shape factor S is

$$S = \frac{Lb}{2t(L+b)} \qquad \ldots 31$$

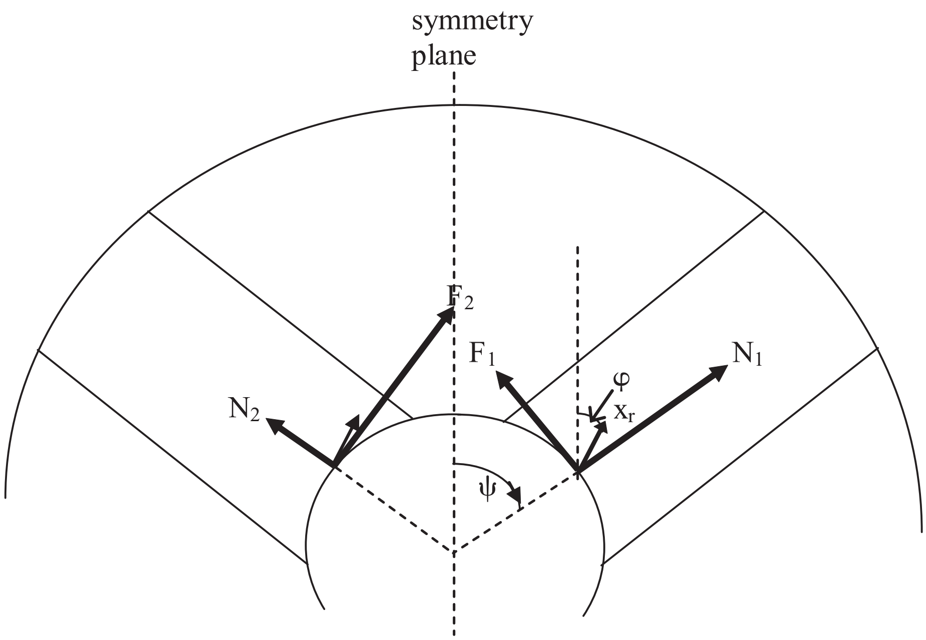

Figure 5. Diagram of shear block mounting for radial deflection.

The stiffness is then given by

$$
\begin{aligned}
k_r(\varphi) &= \frac{1}{x_r}\left[N_1\cos(\psi-\varphi)+N_2\cos(\psi+\varphi)+F_1\sin(\psi-\varphi)+F_2\sin(\psi+\varphi)\right] \\
&= \frac{GLb}{(r_o-r_i)}\left[3\left(1+\frac{L^2b^2}{2(r_o-r_i)^2(L+b)^2}\right)\left(\cos^2(\psi-\varphi)+\cos^2(\psi+\varphi)\right)+\sin^2(\psi-\varphi)+\sin^2(\psi+\varphi)\right]
\end{aligned}
\quad \ldots\ 32
$$

Combined compression and shear of rectangular bonded rubber blocks for finite deformations has been considered by Hill and Myers[14]. Only a plane strain deformation, applicable to very long block, was considered and planes initially parallel to the bonded ends remained planar and parallel to the bonded ends. The stiffness equations derived by Hill and Myers were evaluated at small strain and compared with a modified version of *Equation 32* with $S = 4G(1 + S^2)$, applicable to long blocks[9].

FEA

FEA models for mountings with various dimensions were set up and run on MSC.Marc 2005. A typical mesh cross-section is shown in *Figure 6*. Symmetry was exploited where possible and the stiffnesses were obtained from the reaction force or moment from a deflection of the inner sleeve of 0.1mm or 0.01 radians. In all cases, the rubber was modelled as a neo-Hookean material with a shear modulus of 1MPa and a bulk modulus of 2000MPa.

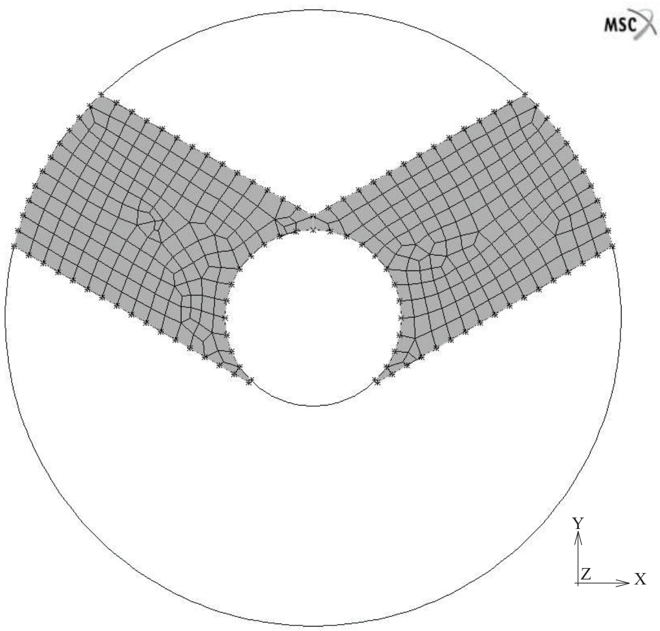

Figure 6. Typical FEA mesh.

Fully integrated first-order Herrmann elements were used. Initial stiffnesses for axial, torsional and radial deformations were obtained. The radial stiffness was obtained in various radial directions. For some mountings the radial and torsional stiffnesses were obtained for both 3D and plane strain models.

RESULTS

The axial and torsional stiffnesses are given in *Tables 3* and *4*. The radial stiffnesses are given in *Tables 5* and *6*.

Discussion

Comparison of the calculations and the FEA results indicate that the axial stiffness is adequately represented by *Equation 22*. The estimation of the torsional and radial stiffnesses from the calculations is reasonable, bearing in mind that they are very approximate. The treatment for the bush in torsion apparently underestimates the stiffness, particularly when bending is taken into account. The calculation of radial stiffness ignores bending and this may be the reason why *Equation 32* overestimates the stiffness for radial deflections

TABLE 3. AXIAL STIFFNESS OF SHEAR BLOCK MOUNTINGS

Dimensions of mounting. The inner radius, r_i = 10mm in all cases.				Axial stiffness N/mm	
Length L mm	Outer radius r_0 mm	Width of one block B mm	Angle ψ degrees	Equation 22	FEA
20	35	20	60	21	20
40	20	20	60	157	151
40	35	20	60	57	52
40	50	20	60	30	28
80	35	20	60	124	115
40	35	10	60	28	26
40	35	20	90	57	52

TABLE 4. TORSIONAL STIFFNESS OF SHEAR BLOCK MOUNTINGS

Dimensions of mounting. The inner radius, r_i = 10mm in all cases.				Torsional stiffness Nm/radian			
Length L mm	Outer radius r_0 mm	Width of one block B mm	Angle ψ degrees	Equation 24 ignores bending	Equation 27 includes bending	FEA (3D)	FEA (plane strain)
40	20	20	60	32	30	35	36
40	35	20	60	22	16	22	23
40	50	20	60	20	11	17	18
40	35	10	60	11	4.6	6.4	6.7
40	35	20	90	22	16		23

TABLE 5. RADIAL STIFFNESS (N/mm) AS A FUNCTION OF DIRECTION FOR A MOUNTING OF LENGTH 40 mm, INNER RADIUS 10 mm, OUTER RADIUS 35 mm AND ANGLE OF INCLINATION OF THE BLOCKS, Ψ OF 60º

Angle of stiffness measurement φ degrees	Equation 32	FEA (3D)	Hill & Myers[14]	Modified Equation 32 (plane strain)	FEA (plane strain)
0	103	87	106	122	102
45	142	135	150	180	164
60	161	158	171	210	195
90	180	182	191	239	225
135	142	134	147	180	163
180	103	87	106	122	103

TABLE 6. RADIAL STIFFNESS FOR SHEAR BLOCK MOUNTINGS

Dimensions. All mountings are of length 40mm and inner radius 10mm		Angle of stiffness measurement	Radial stiffness N/mm			
Outer radius r_o mm	Angle of inclination ψ degrees	φ degrees	Equation 32	FEA (3D)	Hill & Myers[14]	Modified Equation 32 plane strain
20	60	0	347	306	402	440
20	60	45	533	477	650	720
20	60	60	627	542	771	860
20	60	90	720	598	880	1000
50	60	0	62	44	63	73
50	60	45	83	76	86	105
50	60	60	94	88	97	121
50	60	90	105	99	108	138
35	90	0	64	39	64	64
35	90	45	142	153	148	180
35	90	60	180	185	191	239
35	90	90	219	213	233	297

along the $\varphi = 0$ direction. Virtually identical results are obtained for $\varphi = 0$ and $\varphi = 180°$, as required at small strain to ensure continuity at the origin.

Comparison with the plain strain FEA suggests that *Equation 32* tends to overestimate the radial stiffness whereas Hill and Myers' treatment underestimates it. Blocks of the dimensions considered here are insufficiently long to approximate well to plane strain behaviour so use of *Equation 32* for approximately square blocks is preferred.

CONCLUSIONS

Approximate calculations have enabled useful estimates of the axial, torsional and radial stiffnesses of voided bushes and shear block mountings to be made, as shown by comparison with FE simulations. The calculations often included quite crude approximations of the bush geometry and did not consider in any detail end and edge effects due to the bonded and free surfaces. This was necessary to keep the mathematical treatment reasonably straightforward.

The widespread availability of FEA codes has made calculation of the stiffness of rubber bushes and mountings straight forward; problems which are intractable mathematically due to their geometric complexity are easily solved using FEA, particularly at small strains. The main shortcoming of FEA codes in this context is their limited ability to model accurately the stress-strain characteristics of rubber, particularly when inelastic effects due to strain history and time-dependent behaviour are of interest. This is of course also a limitation of design formulae which generally assume linear elastic behaviour.

The approximate formulae presented here have been programmed into an Excel spreadsheet to enable "first guesses" of stiffnesses to be obtained as an initial step in the design process. FEA may then be used to refine the initial design.

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